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**GRAPHIC METHODS FOR PRESENTING
BUSINESS STATISTICS**

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GRAPHIC METHODS

FOR PRESENTING

BUSINESS STATISTICS

BY

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PREFACE

The fundamentals of graphic presentation are simple to those who will give them a little serious study. Once understood there will be no difficulty in applying them to any particular problem. The principles of presenting business statistics are fairly well standardized at the present time. It is recognized also that a definite practice of business charting has been developed which is distinct from engineering charting though it is closely related to it. It is the object of this book to contribute to the work of standardizing this practice. The discussion is confined to the common forms of statistical graphs, and does not touch upon devices like organization charts, routing charts, and other charts in which the subject matter is not statistical. Unusual forms of statistical charts are also omitted.

As it is assumed that the reader has a general acquaintance with the use of charts, but little space is given to attempting to prove the merits of graphic presentation as a whole. The merits of specific types and forms, however, are carefully considered, for no one can safely use graphic methods unless he clearly understands their specific uses and limitations.

Every essential point made in the text is illustrated but no illustrations are included for which there are not specific purposes. As far as possible the practical use of each form of chart is shown by using illustrations drawn from actual practice, though in many cases it was necessary to redraw the chart and conceal the identity of the source because of the private nature of the data presented.

The text is technical. Yet no technical language has been used which is not fully explained. Mathematical terms which would not be familiar to the business student have been carefully avoided. Having had experience in explaining graphic methods to various business men as well as to about twelve hundred different students who were not mathematicians or engineers, the author believes that the language that is used can be understood clearly by any business man or student who will give it serious study.

This book may be said to be largely a compilation of points which are hidden in or omitted from other works on graphics. During the time that the author was an instructor in the Harvard Graduate School of Business Administration he gave a good deal of attention to the graphic presentation part of the course in Business Statistics. In going through this work with twelve different laboratory classes, a careful record was made of the points that had to be emphasized in the lectures in order to get the best results in making and understanding statistical charts. This work has been continued in university teaching in southern California, and it has been revised and added to considerably in the author's consulting practice.

Due credit should be given to the works of Willard C. Brinton, Allan K. Haskell, G. U. Yule, W. I. King, Horace Secrist, and Thomas E. French, for it is from these sources that the author gained many of the ideas that are presented in this volume.

The author wishes to express his appreciation of the help of the many persons who have contributed to the production of this volume. He wishes to thank in particular Professor Edmund E. Day, who suggested the plan for organizing the material, liberally supplied illustrations from his files, and made many valuable criticisms of the original manuscript, and Mr. M. C. Rorty, who wrote the introduction and made many important improvements throughout the book.

He also wishes to thank Messrs. George J. Eberle, A. J. Hettinger and Thurston H. Ross for reading the manuscript and making many valuable suggestions which were incorporated in the work.

Acknowledgment should also be made to Messrs. William F. French, R. S. Parker, Hillebert W. Smith, John L. Niceley, Charles E. Berry and William M. Holmes for assistance in the preparation of text and charts, and to Inez W. Sparling for faithful services in reading manuscript and proof.

In a work of this kind, which contains so much detail, it is impossible to avoid all errors. Any corrections or criticisms will be welcomed by the author and any suggestions indicating how graphic practice can be further standardized will be especially appreciated.

JOHN R. RIGGLEMAN.

LOS ANGELES, CALIFORNIA
December, 1925

CONTENTS

	PAGES
PREFACE	v-vi
INTRODUCTION BY M. C. RORTY.	xi-xiii

CHAPTER I

GRAPHIC CHARTS IN BUSINESS	1-4
Necessity for Clear Presentation—Visual Appeal—Visual Comparison—Aid to Memory—Use in Analysis—Development—Saving of Time—Study Necessary on Part of User—Specific Uses in Business.	

CHAPTER II

TECHNIQUE OF DRAWING STATISTICAL CHARTS	5-32
DRAWING EQUIPMENT AND ITS USE—Charting Paper—Pencils—The Drawing Board—The T-Square—Triangles—Irregular Curves and Flexible Rulers—Scales—Protractors—Compasses and Dividers—Ruling Pens—Ink—Erasers—Tinting—Mounting and Adhesives—The Section Liner—Optical Illusions—Copying Devices.	
LETTERING—Styles of Letters—Hand Lettering—Lettering Instruments—General Proportions—Free-hand Single Stroke Lettering—Vertical Single Stroke Capitals—Vertical Lower Case—Inclined Capitals—Inclined Lower Case—Common Errors to be Avoided—Large Commercial Gothic—Handmade Modern Roman—Spacing—Mechanical Lettering—The Drafting-room Printing Press—Rubber Stamps—Pasting on Printed Titles—Cut-out Letters—The Typewriter.	

CHAPTER III

PRINCIPLES OF CHART PLANNING	33-41
Purpose of the Chart—Size and Proportions of the Chart—Printed Forms for Charting—Standardization of Charts—Degree of Accuracy in Charting—Position and Size of the Title—Importance of a Clear Title—Use of Color—Pictorial Charts—General Appearance.	

CHAPTER IV

SIMPLE COMPARISONS OF SIZE.	42-79
Choice of Geometric Forms for Simple Comparisons of Aggregates—Pictorial Forms—Bar Charts—Vertical and Horizontal	

Bars—Bar Chart Scales—Guide Lines to Aid in Reading Scale or Comparing Bars—Designating Bars—Common Errors—Showing Decreases or Losses along with Increases or Gains—Systematic Order of Items—Comparisons of Differences—Common Unit Necessary—Mechanical Bar Chart—Comparisons of the Component Parts of an Aggregate—The Pie Chart—The Component Bar Chart—Complicated Forms Should Be Avoided—Pictorial Forms for Showing Component Parts—Component Parts of Component Parts—Comparisons of Aggregates Made Up of Components—A Common Error in Using Component Bars—Checking List.

CHAPTER V

HISTORICAL TIME SERIES. 80-129

SIZE CHANGES OF A SINGLE VARIABLE OVER A PERIOD OF TIME—Simple Pictorial Time Charts—Bar Charts for Time Series—Curves of Size Changes—The Arithmetic Vertical Scale—Importance of Including Zero on the Vertical Scale—The Horizontal Scale—Emphasis of Lines—Scale Units and Number of Coordinate Lines—Showing Plus and Minus Quantities—Smoothed and Broken Curves—Step Curves—Polar Coordinates—Presenting Tabular Data on Charts.

COMPARISONS OF SIZE DIFFERENCES BETWEEN TIME VARIABLES—Choice of Forms—Bars and Curves—Scales for Size Comparisons—Differentiating Curves—Designating Curves—Number of Curves—Standardization—Range of Variations—Double Time Scales—Different Time Scales—Smoothed and Broken Curves in Comparisons—Shading to Show Differences—Indicating Forecasts.

COMPONENTS OF A TIME VARIABLE—Choice of Forms—Bars and Curves—Designating Shaded Areas—Actual Sizes and Percentages—A Common Error—Shading Components—Checking List for Curves.

CHAPTER VI

HISTORICAL TIME SERIES (CONTINUED). 130-154

RATES OF CHANGE IN A TIME VARIABLE—Simplicity of the Logarithmic Curve—Scales for Logarithmic Curves—Semi-logarithmic Paper and Ruling—Test for the Logarithmic Scale—The Logarithmic Scale Has No Base or Zero Line—Significance of Slopes of Curves—Smoothing Logarithmic Curves.

COMPARISONS OF RATES OF CHANGE IN MORE THAN ONE VARIABLE—Only Vertical Distances Should Be Compared—Scale Units and Designations—Common Units Not Necessary in Comparing Rates of Change—Comparing Data of Unlike Time Distribution—Standardization—Shifting Curves for Ease of Comparison—Percentage Relatives—The Simple Percentage Method—Standard Deviations.

CHAPTER VII

	PAGES
FREQUENCY DISTRIBUTIONS	155-171
Scales for Frequency Graphs—Class Intervals—Designating Class Intervals—Continuous and Broken Series—Smoothed and Broken Curves in Frequency Graphs—Smoothing a Frequency Curve—Cumulative Frequency Curves—Comparisons of Frequency Distributions.	

CHAPTER VIII

CORRELATION	172-180
Primary Correlation—Secondary Correlation—Historical Time Series and Frequency Series Distinguished from Other Correlative Series—Smoothed and Broken Curves—Position of the Curve—Comparison of Correlation Curves—Use of the Logarithmic Scale.	

CHAPTER IX

GEOGRAPHICAL DISTRIBUTIONS	181-212
Outline Maps—Simple Comparisons of Geographical Areas and Distances—Simple Distribution of Size—Methods of Representing Numbers—Magnitudes at Geographical Points—Radial Distances—Representing Sizes for Certain Areas—Representing Size by Shading—Densities—Geographical Distributions Where Something Other Than Area is a Factor—Use of Colors in Statistical Mapping—Comparisons of Geographical Distributions—Tables Should Accompany Maps—The Map-tack System—Map Mount—Map Marking Devices.	

CHAPTER X

DUPLICATION, REPRODUCTION, AND DISPLAY	213-222
Blueprinting—Drawing for Photographic Reproduction—The Photostat—Cuts or Plates for the Press—Ben Day Shading—Mimeograph and Hectograph—Lantern Display—Display Fixtures.	
INDEX	223-231

INTRODUCTION

An eminent scientist has defined language as "any code of signals capable of communicating information or thought." On this basis he has asserted that a language might be devised in terms of any one of the physical senses. The language of touch might be nothing more than the Morse telegraph code transmitted by means of a series of finger taps; or it might be the embossed type used by the blind. The languages of taste and smell might equally well be constructed in the form of arbitrary codes of alternating and successive flavors and odors. And the languages of sight and hearing, as our everyday experience tells us, are nothing more than arbitrary codes of letters and sounds.

In whatever manner language has developed, it has gained in breadth and flexibility and comprehensiveness as it has become more arbitrary in form—but, at the same time, it has lost in vividness. The upraised hand of the traffic policeman speaks in a language clearer even than that which he uses when his signal is disregarded. A hand clasp, a smile, or a frown may say more than any written page.

The outstanding factor in all language, in all the varied ways in which we perceive our environment and communicate with it, is the predominant influence and importance of the visual sense. The greater part of our memories, and perhaps 90 per cent of our constructive thinking, is in terms of visual images. The man who can most successfully plan a campaign for the future is that one who can most clearly set the stage in his own mind for the play that is contemplated, and can make his actors carry out the phantom drama to the end without missing an essential move or gesture. Those who have this power of exact thinking in terms of visual images are usually unconscious of the methods by which they arrive at their conclusions. Many great mathematicians and physicists work out rough solutions of their problems by these visual methods before they apply the final check of mathematical formulae; but they rarely can describe the type of imagery which they employ.

This general unconsciousness of the extent to which thought is expressed in terms of visual images is, however, the best of evidence as to the pervasiveness of the sense of sight in our mental processes. We *grasp* a simile to *strike* the imagination of a reader until his thoughts *run* with ours, and each word italicized represents a primitive visual image stolen from its original sense for a broader use.

With this fundamental background in human habits and experience, no apology is necessary for the attention which has been given in recent years to the study of graphics as an aid to statistical presentation. The old facts of business and human experience are valueless except as they lead to new action, sanely conceived. Such facts must, in the main, continue to be presented on the printed page—but on such pages the cold figures of a balance sheet, or the massive columns of a census report, speak in clear tones only to the expert who has the time to read them with care. For the ordinary business man, or even for the busy expert, the typed page must, therefore, have its supplemental language—a language which will help to bridge the gap between the arbitrary typed symbols of words and figures, and those mental images and that clearness of comprehension which must be created before action can ensue. This supplemental language is the language of graphics—and the essential and somewhat unique thing which Mr. Riggleman has done is to teach the language of graphics without at the same time unnecessarily confusing his reader by attempting to teach the science of statistics.

The distinction between the language of graphics and the science of statistics is as well marked as that between the mere ability to read and write and the special skill of a trained journalist. And just as the ultimate star reporter must begin his career by learning the alphabet, the vocabulary, and the grammar of the language in which he is to write, so also there is much advantage for the beginner in statistics, and in many cases for the trained or naturally gifted statistician, in studying the alphabet and grammar of graphics as a thing apart from the elaborations of statistical technique.

Mr. Riggleman has also done another thing which is worthy of remark. He has built his book out of copious notes drawn from practical experience in the teaching of graphical methods to both college students and business men. This is the surest guarantee of the practical usefulness of the volume.

To say this much of graphical presentation in general, and of Mr. Riggleman's book in particular, is not difficult, but it is less easy to accede to the request that the writer should illustrate his point of view by specific suggestions arising from his own experience. There is little that can properly be included in the language of graphics which the present text has left uncovered. Any amplification of the subject must, therefore, be in the direction of indicating the next steps that naturally will be taken by the student after he has mastered the language. In this respect, perhaps, the greatest principle in business statistics is that every single presentation to the executive, whether graphical or otherwise, should carry its definite message. This message may be "rest content with the situation as it is"; or it may be "investigate in further detail"; or it may be a call for definite action. But in every case some message must be evident. There is no greater confession of weakness in business statistics than for the executive to say, "Very interesting—but what shall we do about it?" and for the statistician to answer, "I do not know."

A second important principle is that of approaching a difficult or confused problem piecemeal, and step-by-step, without insisting that the entire method of solution shall be evident from the beginning. In such cases it is always possible to select one or more facts, or whole series of data, which must obviously be assembled before any answer is possible. When these facts have been gathered together, plotted and analyzed, they will quite certainly suggest the need for further facts. And, sooner or later, if this process is repeated, the direct road to a solution will present itself.

The two principles just stated are, perhaps, the most general that apply in business statistics. For this reason it may not be inappropriate to mention them as an introduction to a volume which is characterized by the simplicity and directness of purpose that are so evident in the pages that follow

M. C. RORTY.

NEW YORK CITY

GRAPHIC METHODS FOR PRESENT- ING BUSINESS STATISTICS

CHAPTER I

GRAPHIC CHARTS IN BUSINESS

Probably ninety per cent of the answers "yes" or "no" given by the business man are based on opinion rather than fact. Even if he has them at hand, the average executive cannot read or analyze columns of figures and masses of data rapidly enough to base his decisions upon them when, as is commonly the case, he is forced to decide quickly.

Most of the fundamental facts pertaining to business activities are capable of measurement, and millions of dollars are spent annually in the collection of data to be used in analyzing business. Usually, however, when the measurements have been made and recorded it is difficult to appreciate the interrelations, or if the data have been carefully studied it may be almost impossible to show another person that the conclusions reached are correct and sound. It often happens in a meeting of a committee or board of directors that a sound and carefully prepared plan is rejected simply because the facts presented are not clear to the opposing side. Though it is important to have accurate and significant data, one cannot get the most valuable results unless the methods of presentation are effective.

If these magnitudes, changes, or distributions are presented in tables or written description, it is not easy for the average mind to grasp or appreciate their relations. However, by taking advantage of the situation that any business fact which can be measured can be presented graphically by means of diagrams or charts, we are able to appeal to the mind through the eye, by which organ man easily compares, either consciously or subconsciously, lengths, areas, and volumes.

It is a fact, common to the experience of everyone, that if we wish to get a conception of a certain space we can do so much

more easily if we are able to make a direct visual comparison of the unknown magnitude with a known magnitude. Thus, it is much easier to estimate the height of a wall if a man of known height stands beside it. Based on the same principle, an impression of this comparison may be conveyed to another person by means of a picture of the objects viewed. Figure 51 illustrates this method. Nearly everyone has a good idea of the size of a passenger train and can picture it in his mind's eye. The great size of the *Aquitania* is clearly shown by picturing an eight-car train upon her deck.

Not only are charts valuable for use in presenting business statistics but they are valuable aids to the memory. The great majority of people can remember the shape of a curve much more easily than they can the variation in a column of figures. The shape of a curve will give directly a mental picture of the changes in the cost of living, for instance, or one's income over a certain period, and these visual impressions are more clearly remembered by most persons than actual figures.

One of the greatest values of the graphic chart is its use in the analysis of a problem. Invariably a chart brings up many questions which require careful consideration and often further research before a satisfactory conclusion can be reached. A properly drawn chart gives us a cross-section picture of the situation. It will not bring out new facts but it will bring out hidden facts.

While charts bring out the hidden facts in a table or mass of data, they cannot take the place of careful analysis. In fact, charts may be dangerous devices when in the hands of those unable or unwilling to base their interpretations upon careful study. This, however, does not detract from their value when properly used as aids in solving statistical problems.

Engineers and technical men have long realized the importance of graphic charts, but it is only recently that their value has been recognized by business executives, salesmen, advertising men, bank officials, lawyers, and farmers. Graphic charts have long been thought to be the tools of those alone who are skilled in mathematics, but one needs to have only a knowledge of eighth-grade arithmetic or decimal fractions to use intelligently even the logarithmic or ratio chart which is considered so formidable by those unfamiliar with it.

In many up-to-date plants at the present time executives and managers value their charts as highly as engineers value their drawings. Daily reports are presented in graphic form for comparison, and at a glance one can appreciate the state of affairs in his business. To most persons columns of figures are dry and uninteresting even though their study is vital to the success of the firm. What the executive desires is a presentation of this material in a form that will permit him to grasp easily and quickly the essential points, and graphic charts are a means to this end.

If graphic methods are to be most effective, it is necessary that those to whom they are unfamiliar give some attention to their fundamental structure. As simple as they are, their use and interpretation will be weak if they are merely accepted on faith. For instance, one is not likely to read an arithmetic chart correctly unless he also appreciates the significance of the logarithmic chart. Yet men stumble blindly along, content to fall into many simple pitfalls, because they feel that mastery of graphic methods is some kind of black magic. This, however, is the fault of the man and not of the method. The past few years have witnessed a great increase in the use of graphic methods by progressive firms throughout the country, and the success with which they are being used augurs well for their future.

Graphic methods are used for presenting all kinds of statistical data. They may be used extensively as part of a well-developed statistical department, or to a limited extent where a few important facts are recorded by a bright clerk or secretary. Graphic records may be kept of the internal conditions of a firm, and along with these the executive may directly compare external conditions with the situation in his own business.

These methods furnish, for example, an excellent means for keeping a perpetual inventory which will show at a glance the value or amount of material on hand, how much is added and taken away, and when the danger point is reached. The credit manager finds them valuable in recording a customer's condition, especially his ratio of liquid assets to current liabilities with reference to their position in the business cycle. Working hand in hand with the credit manager, the collection manager can determine at a glance the financial standing of the customer and whether or not it is possible for him to live up to the terms with the company. In studying costs there is a graphic charts. It is in this connection that

charts present some of their most surprising and important revelations. There is almost no limit to the use of charts in analyzing labor statistics such as labor turnover, causes of accident, or payment plans. In the sales and purchasing departments charts are used to keep the manager in touch with the progress of events both inside and outside the firm. The study of financial statistics is greatly aided by charting the data to show amounts and rates of change.

One of the most important uses of the graphic chart is in forecasting earnings, expenses, sales, production, etc., for periods of from 1 to 5 years (revised periodically), and then in comparing actual and forecasted results. The graphic method used in this way, in connection with each element of business as well as the business as a whole, is one of the most effective of all methods of supervision.

These uses and many others will be suggested to the reader by the occasions which arise if the fundamentals presented in the following pages are carefully studied and assimilated.

CHAPTER II

TECHNIQUE OF DRAWING STATISTICAL CHARTS¹

The actual drawing of statistical charts is not an art which should be learned only by the comparatively few who intend to become professional makers of them, but it should be understood by all interested in business statistics, because it gives valuable training in quick and accurate reading of data presented in the graphic language.

From the point of view of making charts of business statistics, this chapter on the technique of chart making is designed to cover: (1) a description of drawing instruments and materials and their uses; and (2) hand and mechanical lettering.

DRAWING EQUIPMENT AND ITS USE

In laying out and drawing charts, various kinds of instruments are used, and in deciding the form in which the data are to be presented, the ability to use these instruments in a correct and efficient manner is quite necessary.

In the selection of drawing equipment one should secure the best that can be afforded, for success in charting depends to a considerable extent upon the quality of the instruments and materials used. Inferior equipment is an annoyance, and continually distracts attention from the actual work in progress, while good equipment will, with practice, leave the mind free from attention to its use and add an environment which is conducive to high-grade work.

The two requirements in the use of instruments are *accuracy* and *speed*, and in drawing charts of business statistics one is as important as the other, and neither is worth much alone. Accuracy must be insisted upon at the outset, and speed with greater accuracy comes with practice. It is natural, however, for the beginner to waste a great deal of time because of the strangeness

¹ Credit should be given to Thomas E. French and his "Engineering Drawing" for many of the ideas on instruments and lettering presented in this chapter.

of his instruments and over-carefulness, so the requirement of speed must receive early attention.

The following list includes the instruments and materials necessary for ordinary work in graphic presentation of business statistics.¹

LIST OF INSTRUMENTS AND MATERIALS

Set of drawing instruments in case, including compasses with needle point, pen, pencil, and lengthening bar; dividers; bow instruments; and ruling pens.	Protractor. Lettering pens and penholders (as- sorted sizes). 2H and 5H drawing pencils. Art gum, erasers, and erasing shield.
Drawing board.	Thumb tacks.
T-square.	Paper.
45 and 30-60 degree triangles.	Other equipment which may be
French or irregular curves and ship curves.	included is mentioned in the description.
12-inch scale (statistician's scale pre- ferred).	

Charting Paper.—Selection of paper depends upon the use to which it is to be put and the personal taste of the individual, but in general it is best to avoid cheap grades. A good paper should be strong, of uniform thickness and surface, should stretch evenly and lie smooth, and allow considerable erasing without spoiling the surface. Cream, buff, and green papers are much easier on the eyes than white paper.

For high-class and lasting work Whatman's Paper and Reynold's Bristol Board are considered to be among the best. They are standard papers and are kept in stock by all well-supplied dealers. Whatman's is made in three finishes: "hot pressed" for fine lines; "not hot pressed" for ink or color; and "rough" for water colors. Bristol board is a smooth paper for fine line work and is commonly sold in three different thicknesses. For most business graphic work many less expensive papers are quite satisfactory if they take ink and color well and stand a reasonable amount of erasing. Bond and ledger papers are also used to a considerable extent in chart making.²

¹ Additional suggestions may be found in catalogues of instrument houses such as Theo. Alteneder & Sons, Philadelphia; the Keuffel & Esser Co., New York; and the Eugene Dietzgen Co., Chicago.

² Detailed descriptions of drawing paper with sizes and prices are given in catalogues of dealers in engineers' and draftmen's supplies.

Pencils.—Lead pencils are graded according to their hardness which is expressed by numbers with the letter H.¹ Personal preferences vary, but for general use in statistical charting a 3H, 4H, or 5H pencil, sharpened to a long conical point, is the most satisfactory. A softer pencil, 2H or H, should be provided also for lettering. The chisel or wedge point is preferred by some for hard pencils, as it does not wear away as fast as the conical point. Pencils may be sharpened to long conical and wedge points by removing the wood with a knife and shaping the lead on sandpaper. Many kinds of special pencil sharpeners have been provided for the draftsmen who care to use them.

The Drawing Board.—Drawing boards and drawing table tops are usually made of well-seasoned, clear white pine cleated at the ends to prevent warping (Fig. 1). The ends should be

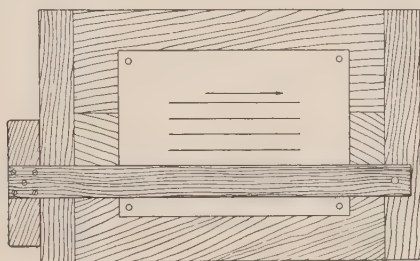


FIG. 1.—Use of drawing board and T-square.

perfectly straight in order that the T-square can be used accurately.

Paper is fastened to the drawing board by means of thumb tacks. It is usually cut larger than the drawing to be made, and should be placed on the board squarely, with the aid of the T-square, and the tacks pushed in up to the head in order that the T-square will not be obstructed.

The T-Square.—T-squares are usually made of hard wood, the better grades having the edges of the blades lined with ebony

¹ Pencils commonly used are harder as the number increases, and are graded from H to 9H. Very soft pencils are graded by their softness which is designated by the letter B and a number which increases as the lead becomes softer. Thus (according to Keuffel & Esser) from soft to hard they are 6B, 5B, 4B, 3B, BB, B, F, HB, H, HH, 3H, 4H, 5H, 6H, 7H, 8H, 9H. Different companies, however, vary in the use of this system.

or celluloid. They are used principally for drawing parallel lines horizontally and for keeping the triangle in position for drawing other lines. Horizontal lines should always be drawn from left to right, and the T-square should always be used from the left edge of the drawing board (Fig. 1) as the edges are seldom parallel or at right angles to each other.¹

Triangles.—Triangles are made of various substances, but transparent celluloid is the most popular. Vertical and sloping lines are drawn by setting the triangle against the T-square and drawing from bottom to top, as illustrated in Fig. 2. Various

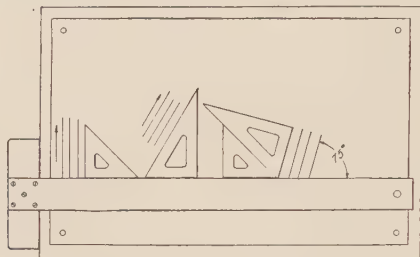


FIG. 2.—Use of triangles.

sizes are made, but a 6- or 7-inch triangle will be found convenient for most chart work, especially when $8\frac{1}{2}$ -by 11-inch paper is used. As celluloid triangles warp easily, care should be taken that they lie flat or are hung up when not in use.

Irregular Curves and Flexible Rulers.—French or irregular curves and ship curves are made of several kinds of material, but transparent celluloid is the most satisfactory. They are laid out in combinations of spirals, ellipses, and other curves such as will closely approximate those commonly met with in practice. A smoothed curve is drawn on a statistical chart by first locating all the points in the series and then so placing the irregular curve that it coincides with several of them. It is shifted from one position to another as the line is drawn. It is usually best to sketch through the points a free-hand pencil curve that is satisfactory to the eye, and then apply the irregular curve, selecting a part that will most nearly fit a portion of the line (Fig. 3). It is very important that the draftsman indicate in advance on his chart the points where the curvature changes. He should then be careful to reverse his curve at such points. Also

¹ This would be reversed for a left-handed man, who would draw from right to left and use the T-square on the right edge of the board.

it is important to select the right type of curve in accordance with the line to be drawn, and to shift this curve so as to get a long fit rather than a short one. When a fit has been obtained, the curve should be used only in its central portion, or portion of best fit, and then it should be shifted and a new fit obtained.

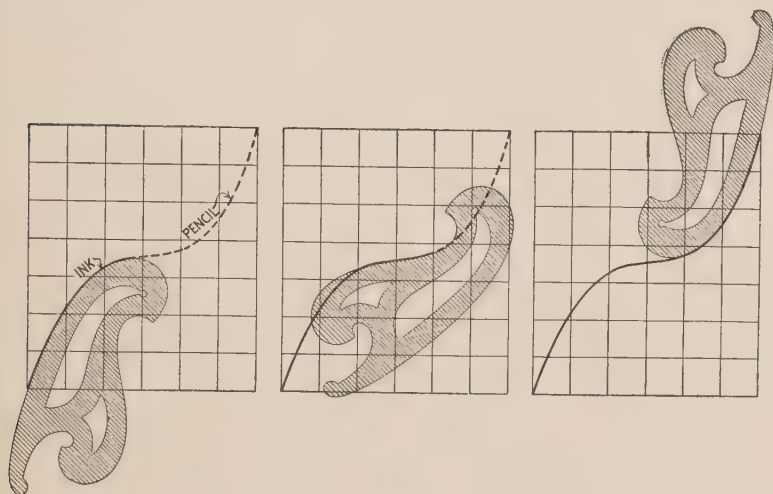


FIG. 3.—Use of French curve. (See catalogues of instrument houses for illustrations of other curves.)

Flexible rulers consist of a ruling edge combined with a soft bar of lead which holds any curve to which the ruler is bent. They are used in a manner similar to that of the irregular curves.

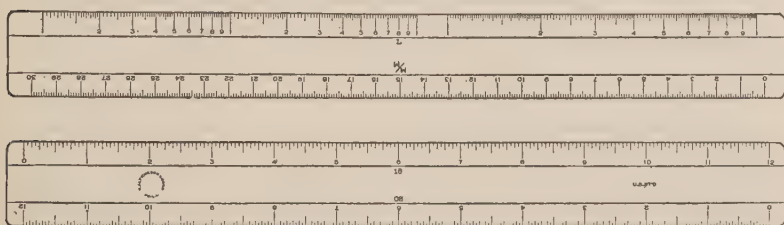


FIG. 4.—A 12-inch statistician's scale.

They can be adjusted, however, to fit a much larger portion of the curve.

Scales.—Statistician's scales are made especially for laying out statistical charts. The one shown in Fig. 4 has four different scales: (1) one in ordinary English rule divisions,—sixteenths of an inch; (2) one in decimal parts of an inch; (3) one in centimeters; and

(4) one logarithmic. This furnishes a scale for making logarithmic graduations, and a choice of three units for making various measurements, laying off spaces, or dividing a given distance into a certain number of equal parts. It has the advantage of having all the scales necessary for chart making in a small and compact form, but it must be tilted in order to bring the edge of the scale into contact with the paper. These scales are made usually of plain boxwood or boxwood with white edges.¹

There are two kinds of engineer's scales in common use—known as the "engineer's scale" of decimal parts, and the "architect's

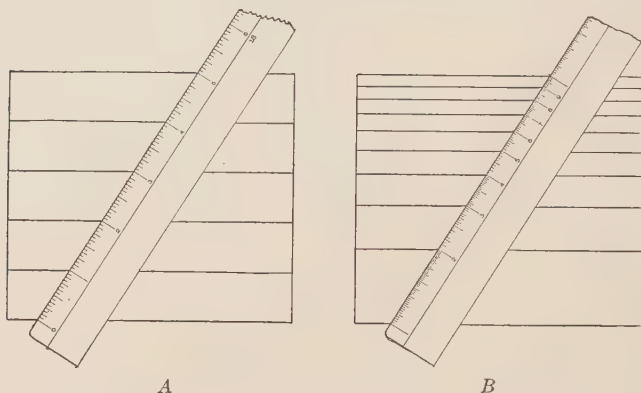


FIG. 5.—A, Arithmetic division of a given space. B, Logarithmic division of a given space.

scale" of proportional feet and inches. They are made in triangular form and also in sets of flat shapes. In chart making these scales are quite widely used, especially by those who have had engineering training.

After the proportions and size of the chart have been laid out, a convenient method of dividing space into any number of equal parts is shown in Fig 5A. The same principle applies to the use of the logarithmic scale (Fig. 5B). If in such a case the logarithmic scale is too short, a line twice the length of the scale can be laid out between the limiting lines, and each logarithmic division doubled as it is laid out along this line.

Another method which is often convenient in using the logarithmic scale, and which may be used also with the arithmetic scale,

¹ Scale Number 2735 of Theo. Alteneder & Sons, Philadelphia, Pa. A triangular scale with logarithmic graduations is made by Keuffel & Esser, New York. Special scales for statisticians can be made to order by scale manufacturers.

is that shown in Fig. 6. It is desired to divide the distance AB logarithmically from 1 to 10 with the scale CD . Place the scale CD a reasonable distance from the line AB and perfectly parallel

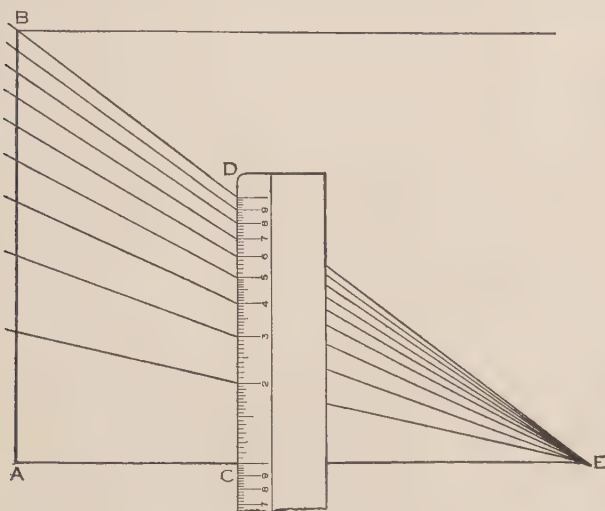


FIG. 6.—Division of a given line.

with it. Locate as many points along CD as are wanted on AB . Draw the lines BD and AC and produce them until they intersect at E . Then lines drawn from E through the points laid off between C and D will locate the logarithmic graduations on the line AB . This method, of course, can be used for logarithmic spacing of a distance either greater or less than the length of the scale used.

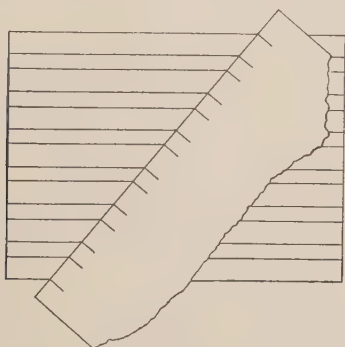


FIG. 7.—Irregular division of a line or space.



FIG. 8.—A protractor.

A convenient method for dividing space as, for instance, in laying out a bar chart where the spaces between the bars are not the same as the widths of the bars, is shown in Fig. 7. With the dividers or scale simply lay out the spaces in convenient units of

the right proportions on a strip of paper somewhat longer than the space to be divided, and use the strip as illustrated in Fig. 7.

Protractors.—A protractor (Fig. 8) is a necessity in laying out "pie charts." The common protractor is a semicircle of 180 degrees, but a more convenient form for pie charting is the percentage protractor made on a basis of 100 parts in a circle. With this form the reading can be made directly from the scale in percentage terms, which is easier and quicker than reading proportional degrees.

Compasses and Dividers.—Compasses and dividers are usually included in sets of various combinations contained in a special case. These combinations vary according to the line of work and means of the user, but should contain at least a pair of compasses with pen, pencil, and lengthening bar, a pair of dividers, and a ruling pen. It is advisable to have an instrument case, for it protects the instruments when not in use and is a convenience when handling the tools, as each one has its special place.

The best instruments are made of German silver and steel, which combine good wearing qualities with ease in keeping clean. The cheaper class is made of brass, which tarnishes easily and is generally unsatisfactory. All modern high-grade compasses and dividers are made with some form of pivot joint, which allows ample movement of the legs and at the same time prevents the occurrence of lost motion from wearing, as is the case with tongue and pin joints.

One leg of the compasses is fitted with a needle point, and the other may be fitted with either a pencil or a pen by means of a shank and socket (Fig. 9). With most instruments the needle is separate and has a square shoulder on one or both ends from which a minute point projects. Before using compasses, this needle point should be adjusted by attaching the pen and setting the needle a trifle longer than the pen (Fig. 10). The needle should be kept in this position and the lead, sharpened

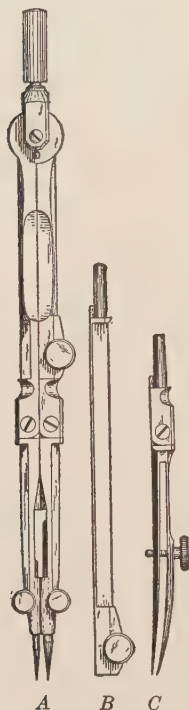


FIG. 9.—A, Com-
passes. B, Lengthening
bar. C, Pen. (Courtesy
of Theo. Alteneder &
Sons.)

to a fine wedge or long bevel edge, should be adjusted to it. This will allow the interchanging of pen and pencil without adjusting the needle point.

When drawing a circle, the compasses, held loosely between the thumb and forefinger, should be inclined slightly in the direction of revolution. Only a light pressure is necessary. In order that both nibs of the pen may touch the paper and only a small center hole be made, it is necessary that the pen and needle be nearly perpendicular to the paper; this is accomplished by bending the legs at the knee joints. For drawing circles larger than the spread of the compasses, the lengthening bar (Fig. 9B) must be inserted between the leg socket and the pen or pencil.

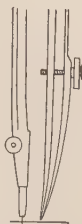


FIG. 10.—Adjustment of needle point.

Dividers (Fig. 11) are used for laying off distances, either from measurements or from other parts of the drawing, and for dividing



FIG. 11.—Hair-spring dividers. (Courtesy of Theo. Alteneder & Sons.)

lines into equal parts. When laying off equal distances the dividers should be turned in opposite directions each time, in order that the instrument can be operated readily with one hand.

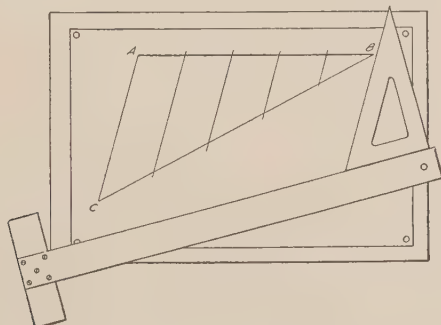


FIG. 12.—Division of a line with dividers.

A form of dividers known as the "hair-spring" is equipped with a screw adjustment in one leg for making fine settings. Combination instruments are made which may be used both as compasses and dividers simply by interchanging the pen or pencil point for a needle point.

A convenient method for dividing a line into any number of parts is illustrated by Fig. 12. To divide line AB into, for instance, five equal parts, draw any line BC , and on it step off with the dividers five convenient lengths. From C , the last point, draw a line to A . Then draw lines parallel to AC through the points on BC , intersecting the line AB which will be divided

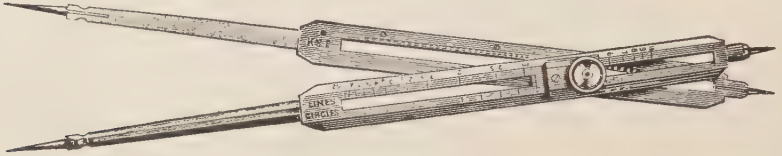


FIG. 13.—Proportional dividers. (*Courtesy of Keuffel & Esser Co.*)

thereby into five equal parts. Triangle and T-square may be used as illustrated for drawing parallel lines.

Another form of dividers, ordinarily not included in sets, is the proportional dividers (Fig. 13) which are used for enlarging or reducing charts in any proportion. The subdivisions marked "lines" are for linear proportions, while those marked "circles"

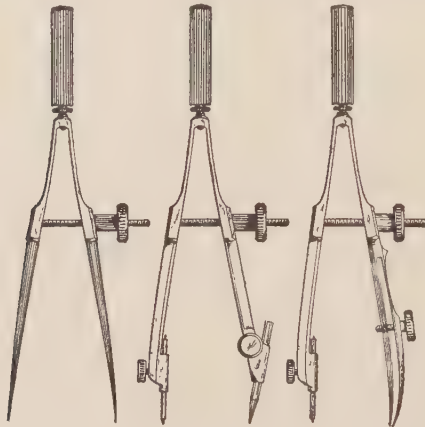


FIG. 14.—Bow instruments. (*Courtesy of Theo. Alteneder & Sons.*)

are for dividing into any number of equal parts a circle whose diameter is measured by the large end.

The bow instruments are used as adjuncts to the compasses and dividers and have their special field of usefulness in making circles and spaces which are so small that it is difficult to make them with the large instruments. The set of three spring bow instruments

includes bow dividers or spacers, bow pencil, and bow pen (Fig. 14). Since it holds its adjustment, the bow pen is almost indispensable in duplicating a large number of small circles.

The drop pen (Fig. 15), with which small circles can be made much faster than with the ordinary bow pen, is of particular convenience in making statistical maps in which small circles or large dots are used. It is held with the needle point stationary and the pen revolving around it.

Ruling Pens.—Ruling pens are made in a number of forms, but all consist of a handle bearing two spring nibs which have a thumbscrew adjustment for regulating the width of line (Fig. 16). Some forms (Fig. 16*B*) have a device which allows the blades to be opened for cleaning, independently of the thumbscrew. For large work a very valuable pen is the form known as the detail pen (Fig. 16*C*).

There are three forms of special pens (dotting pen, curve pen, and railroad pen) which are of occasional value in drawing curves, though they are not required in ordinary work. The dotting pen may be used when making several distinct curves on a chart; it will make several different kinds of dotted or dashed lines successfully when carefully handled. The curve pen is made with a swivel for curve drawing, and the railroad pen is used for making double lines.

The ruling pen is filled by placing the ink between the nibs with a common steel pen or with the quill filler or dropper that is

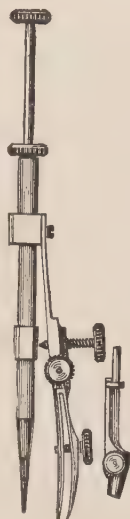


FIG. 15.—
Drop pen.
(Courtesy of
Theo. Alteneder
& Sons.)

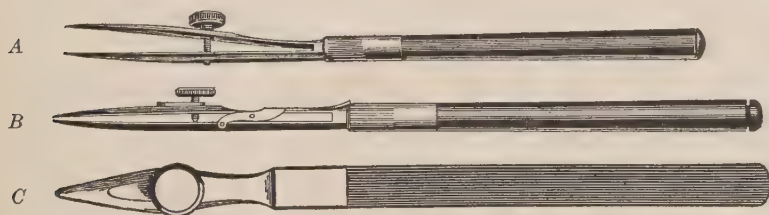


FIG. 16.—Ruling pens. (Courtesy of Theo. Alteneder & Sons.)

provided with most drawing inks. The ink should fill the pen to a height of from $\frac{3}{16}$ to $\frac{1}{4}$ inch depending upon the size of the pen; if too much is used its weight will cause it to run out upon the

drawing. The width of the line is varied by changing the distance between the pen points with the thumbscrew.

The ruling pen is never used free-hand, but always with a guiding edge. It should be held with the blades parallel to the edge, the handle slightly inclined in the direction of motion, and in a plane which passes through the line perpendicularly to the paper (Fig. 17).

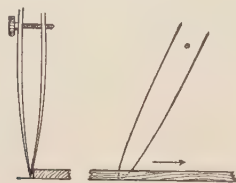


FIG. 17.—Correct position for using ruling pen.



FIG. 18.—Pen blade corroded by ink.

Ink will dry rapidly in the point of the pen when it is adjusted for fine lines and may refuse to flow after the pen has remained idle for a few seconds. If gently pinching the nibs or touching the point on the back of the finger does not start the flow, the pen should be cleaned and filled with fresh ink. Pens should be wiped clean when through using, because a dirty pen will not

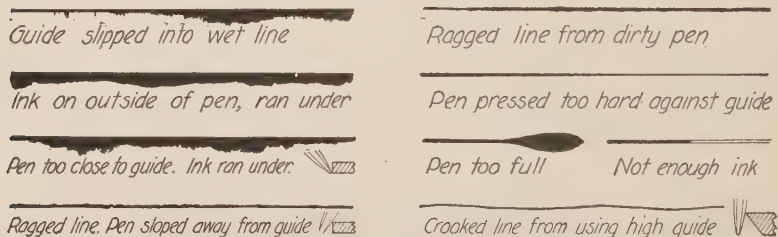


FIG. 19.—Beginners' common faults in ruling lines.

draw clean-cut lines and ink left in the pen will corrode the steel blades (Fig. 18).

Inked lines may be imperfect because of the fault of the pen, the ink, the paper, or the draftsman. Figure 19 shows several common faults for which the draftsman is to blame.

If a pen has become worn, or is too sharp or uneven, it can be put in proper condition by grinding on an oilstone. The points should be given first the proper shape (Fig. 20) by screwing

them together and drawing them back and forth over the stone. Then the pen should be opened slightly and the *outside* of each blade ground by giving the pen a rocking motion at a small angle with the stone. One must be careful not to alter the shape of the points or to grind them too sharp. They should be sharp enough to draw a fine hair line, but not sharp enough to cut the paper. If a burr is formed on the inside, it may be removed by touching it lightly upon the stone, but the inside of the blades should never be ground.

Ink.—Waterproof India ink is usually used for chart making, as it leaves a permanent black line. It is composed of finely divided carbon held in suspension, with shellac added to make it waterproof. Non-waterproof ink flows more freely but is not popular because it smudges very easily. A blotter should never be used on drawing ink, as it removes part of the layer of carbon and leaves a very weak line.

Black drawing ink can be secured in sticks, but the more convenient liquid form, furnished in bottles equipped with a quill for filling the pen, is usually used. A handy ink bottle holder and pen filler, which can be operated readily with one hand, is illustrated in Fig. 21.



FIG. 21.—A handy ink bottle holder and pen filler. (Courtesy of Theo. Alteneder & Sons.)

When not in use, ink bottles should be kept tightly corked to prevent thickening and drying up. Drawing ink must not be allowed to freeze, as once frozen it is useless.

Waterproof drawing inks are also made in a variety of colors. As colored inks are thinner, they flow more readily than black when used in a ruling pen.

Erasers.—Although but little erasing should be necessary, a good ink eraser should be provided for erasing errors in lines or letters. In using erasers, care must be taken not to damage the paper or the surrounding work. Erasing shields, usually made of thin metal, are used to protect the work around the erasure which is made through one of the slits.



FIG. 20.—Correct shape for ruling pen blades.

Drawings are cleaned by rubbing with a piece of art gum, or sponge rubber. However, a finished drawing should not be scrubbed all over with art gum or rubber, as it takes the life out of the inked lines.

Tinting.—The usual method of tinting is to apply light washes of water colors with a camel's hair or sable brush. As red sable brushes are not affected by water, they are better for this work than camel's hair brushes which lose their elasticity after being wet. These brushes are commonly fitted with long handles which often may be made more convenient by cutting down to a length of about 7 inches. Three brushes the size of those shown in Fig. 22 will be sufficient for ordinary chart work. Brushes should be cleaned when through using, and should not be allowed to stand with the weight of the handle on the hair.

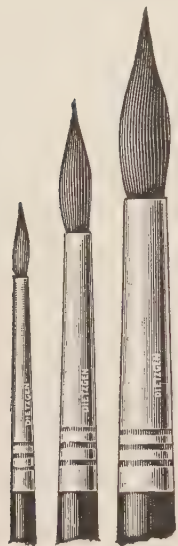


FIG. 22.—Water color brushes—full size. (Courtesy of Eugene Dietzgen Co.)

After the drawing has been inked and cleaned, the part to be tinted should be gone over with a wash of clear water in order that the color will spread evenly. A pool of color is then led with a brush over the surface. After the surface has been covered, the brush should be wiped out and used to pick up the excess color. The tints should be mixed in shallow dishes, as light washes and deep colors can be obtained by repetition of the washes. One should see that the wash is thoroughly mixed and that it is not too thick.

Colored inks may be diluted and used in the same way as water color washes.

Crayons are handy to use and good effects can be secured if they are carefully applied. They should be applied very lightly with a flat point and with horizontal strokes. Depth of color should be secured by repetition of strokes—not by pressure which makes it impossible to get an even color. Various line or dot effects can be secured by placing the drawing over different kinds of irregular surfaces, such as book bindings, and then rubbing with crayons. Some crayon work can be evened by smudging with a cloth over the end of the finger, but usually this process takes the life out of the color. Work done in soft

crayon may be made permanent by spraying on a shellac fixatif with an atomizer.

Mounting and Adhesives.—There is no single method of mounting that can be used for all cases. Different purposes and materials require different adhesives and handling.

A very satisfactory adhesive for temporary mounting in general chart work is ordinary liquid rubber cement, such as that used in repairing automobile inner tubes. It does not warp the paper, and any excess may be rubbed off the chart with the finger or an eraser, leaving the paper clean. Paper mounted with this cement is held securely, but may be pulled off without tearing as the cement is not as strong as paper. As different kinds of cement vary widely, care should be taken, when purchasing rubber cement for use on paper, to select that which sticks well and is light in color. These cements may be spread with the finger or with a brush and should be applied freely over the entire surface of the paper which should be pressed firmly into place. A better method is to coat both surfaces, letting them dry a few minutes before pressing together. Where great strength is desired, as in joining paper with a small lap, these cements are not practical, and for permanent mounting they are unsatisfactory as they lose their adhesive qualities with age.

If paper is stretched on a drawing board and the edges fastened with glue, another paper can be mounted on it with thin photo paste or liquid glue without warping. It must, however, be allowed to dry thoroughly before being cut from the board. If loose papers are stuck together with paste or glue they will warp but sometimes they can be smoothed out reasonably well with a hot iron.

There are two common methods of mounting paper on cloth—hot mounting and cold mounting. Hot mounting can be done much more rapidly than cold mounting and for this reason is generally used. To hot mount a chart, first stretch the cloth tightly over a cloth-covered board and fasten securely with tacks. Then cover the back of the chart with a thin hot photo paste or liquid glue, brushing from the center outward. Have a hot iron ready and press the chart down upon the cloth, working rapidly from the center toward the edges. As soon as the edges are stuck, remove the tacks and raise to let out the steam. Iron on the chart side until dry. If ironed on the cloth side the steam formed will cause blisters. Cold mounting is done in the same

way except that cold paste and a photographic print roller are used, instead of hot paste and an iron, and the chart is allowed to dry thoroughly before the tacks are removed. Paste and glue

work well in both methods, but glue breaks more easily if the chart is folded.

The Section Liner.—For cross-hatching on bar charts, belt charts, and maps, the mechanically spaced rule, commonly called the section liner, is a valuable aid. It can be adjusted for various spacings, and the ruling edge is moved from one space to another by pressing a small lever.

Optical Illusions.—In cross-hatching or shading areas it must be recognized that one must contend with certain optical illusions. Occasionally these illusions spoil the effect of the entire chart. Some of the most common are

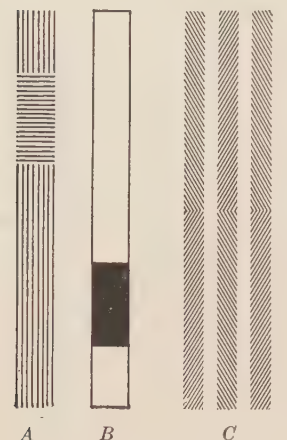


FIG. 23.—Optical illusions.

illustrated in Fig. 23. Note that in *A* the part of the bar shaded with horizontal lines appears to be narrower than the parts shaded with vertical lines. In *B* the part of the bar shaded in black appears to be narrower than the white parts. In both *A*

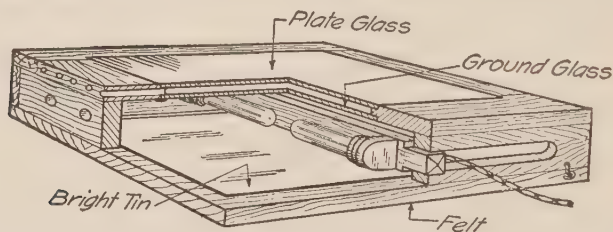


FIG. 24.—Glass board for copying and comparing charts. (From "Engineering Drawing" by Thos. E. French.)

and *B* the sides of the bars are perfectly straight and parallel. In *C* the first and second bars appear to be closer together at the ends than in the middle, and the second and third appear to be farthest apart at the ends while, as a matter of fact, the three bars are perfectly parallel.

Copying Devices.—A convenient device for copying charts on heavy paper is the glass copying board or table. It is so made that natural or artificial light passes through the glass from below

and the opaque lines and letters can be distinguished easily from the more translucent parts. The better forms consist of a plate glass top over a ground glass which evenly distributes the light furnished from underneath by electric lamps mounted over a bright reflector (Fig. 24). This device is valuable also in comparing different charts by superimposing one over the other. As an emergency substitute, a chart may be clipped to the drawing paper and held up to the light against a window.

For transferring the outline of a chart, carbon paper can be used providing the original is not too stiff. The chart and drawing paper, with the carbon sheet between, are so tacked down that one will not move upon the other, and the lines traced with a small round point. A method similar to this is transferring by pricking, in which the chart is laid over the paper and pricked through with a needle point, turning the chart back frequently and connecting points.

Charts can be reduced or enlarged in any proportion by means of the pantograph. It consists of a parallelogram of four bars having the tracer, pencil, and pivot in a straight line. Any adjustment of the bars conforming to this arrangement will copy in true proportion. A simple wooden form is inexpensive and accurate enough for most chart drawing, but better work can be done with the finely constructed, suspended pantograph.

LETTERING

Lettering is closely related to drawing but is not part of it. It is a separate and distinct subject covering the making and use of letters of various accepted styles and sizes. Letters may be made either by hand or mechanically. Hand lettering is the process of making them by the free-hand single stroke method or by the method of drawing them with the aid of drafting instruments. Mechanical lettering involves the use of mechanical equipment such as the draftsman's printing press or the typewriter. As far as appearance is concerned, there is no part of a chart more important than the lettering. A good drawing may be ruined by poor or careless lettering, not only in appearance but in its effect, as any evidence of carelessness or lack of skill tends to destroy confidence in the work.

Two fundamental requirements, which any method of lettering used in graphic presentation of business statistics must possess, are *legibility* and *speed*. If it has these qualities, almost any con-

sistent style of plain, neat lettering is acceptable in the ordinary course of chart work. Ornate lettering has its place among artists and craftsmen, but is not practical or appropriate for business charts.

Styles of Letters.—The most common styles used in charting are the “Commercial Gothic,” and the “Modern Roman” or its modifications (Fig. 25). In hand lettering Commercial Gothic is used chiefly, and in mechanical lettering both styles are used.

OLD ROMAN

Roman lower case

MODERN ROMAN

Gothic

COMMERCIAL GOTHIC

FIG. 25.—A few of the different styles of letters in common use.

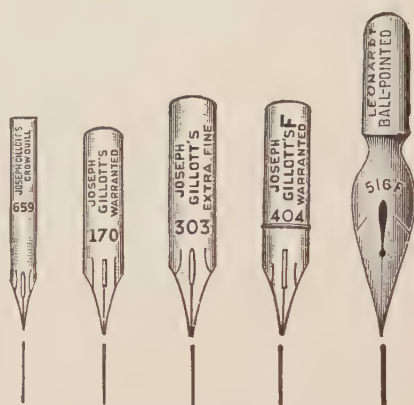


FIG. 26.—Lettering pens

Hand Lettering.—There are two general divisions of hand lettering—free-hand single stroke or written letters, and drawn or built-up letters. Small Commercial Gothic are made by the free-hand single stroke method but the large size are usually drawn with the aid of instruments. Both large and small Roman are commonly made in outline and filled in.

Lettering Instruments.—Many different kinds of steel writing pens are used for lettering. Figure 26 shows several pens which are widely used for making letters under $\frac{1}{4}$ inch high. The size of pen to be used depends upon the width of line desired for the lettering. The pen should be selected which will make the desired width without spreading the point. Fine pens should not have points so sharp that they will scratch the paper and for this reason lithographic pens are not well adapted for chart lettering. New pens are covered with a very thin film of oil which should be removed before using by wetting and wiping dry. Steel

pens may be inked by dipping them into the bottle, but many consider it better to fill them with the quill or dropper. If too much ink is used it has a tendency to run where strokes are joined in the letters (Fig. 27). Lettering pens should be given as good care as other instruments, for a well-broken-in pen is more valuable than a new one. The ordinary cork grip pen-holder is commonly used, though various other kinds are furnished to suit personal preferences.

F H K W X

FIG. 27.—Too much ink.

A very popular form of special pen for making letters from $\frac{1}{8}$ to $2\frac{1}{2}$ inches high is the Payzant pen (Fig. 28). Uniform lines can be made easily with this pen as it is constructed to produce the same width and density of line no matter in what direction the pen is moved over the paper. It is made in various sizes, the Barch-Payzant (Fig. 28B) being made in eleven

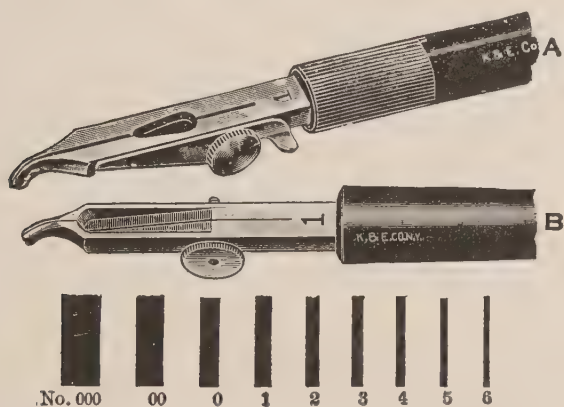


FIG. 28.—Payzant lettering pens. (Courtesy of Keuffel & Esser Co.)

sizes, making lines from 0.012 to 0.2 of an inch wide. The Payzant is filled by placing the ink in the reservoir with a quill or dropper. It should never be dipped into the ink. When the pen becomes clogged, the flow may be started by inserting a piece of stiff paper between the nibs. Strokes made with a Payzant have rounded ends, and if a better finish is desired, sharp corners are added with a fine pen.

Letters larger than $2\frac{1}{2}$ inches high, such as those used in titles of wall charts, are best made by outlining with drawing instruments and filling in, but fair letters may be made much more quickly by using a brush or a pen such as the automatic marking

pen (Fig. 29). When the automatic is used, care should be taken to see that the letters are correctly shaped. This form of pen is made in various sizes for lines from $\frac{1}{16}$ to $\frac{1}{2}$ inch in width. It should always be filled with a quill or dropper and never by dipping it into the ink.

A convenience for making guide lines is the Braddock lettering triangle. The pencil point is inserted through the holes and the line drawn by sliding the triangle along the T-square or other straight edge. The numbers of the columns of holes express the heights of capitals in thirty-seconds of an inch. The vertical or slanting guide lines may be drawn in also with this instrument, using it as an ordinary triangle. Several other lettering triangles

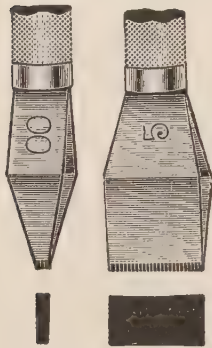


FIG. 29.—Automatic marking pens. (Courtesy of Eugene Dietzgen Co.)

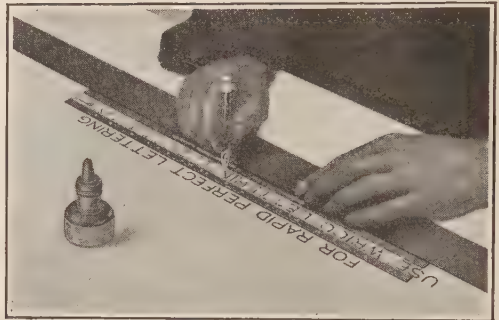


FIG. 30.—Wrico lettering guide. (Courtesy of the Wood-Regan Instrument Co.)

similar to the Braddock are made by the different instrument houses.

After one has studied lettering and knows what sizes to use, and if one understands spacing, the lettering guide illustrated in Fig. 30 can be used as an aid in making more accurate forms. The lettering on the chart in Fig. 206 is an example of work done with the aid of this device. A good form of pen to use with all except the smallest of these guides is the Payzant illustrated in Fig. 28.

General Proportions.—There is no standard proportional relation of width to height in letters. When space for lettering is short, "**COMPRESSED LETTERS**," which are very narrow in width as compared to height, are used. When it is desired to spread a short title over a long space "**EXTENDED LETTERS**"

which are wider than normal, are used. The most legible and stable appearance of capitals is secured by making the width about seven-eighths of the height (except I, M, and W). Letters also vary in the thickness of stroke in proportion to height. Those with heavy strokes are known as "**bold face**" and those with narrow strokes as "light face" letters.

In making letters one must contend with certain optical illusions. Round letters, such as C, O, or Q, will appear to be smaller than square letters, such as E, F, or Z, if they are made exactly the same height. This is true also of letters such as A and V which would appear to be too short at the pointed end. In order to give the appearance of equal heights, the round or pointed ends of letters should be extended a little over the guide lines which limit the letters with square ends. If letters such as E, H, or S are to look well proportioned, their top and bottom halves should appear to be equal. This effect can be secured only by making the letters slightly narrower at the top than at the bottom and drawing the center lines slightly above the middle. This illusion will be quite evident if any printed page is turned upside down and the letter S observed. The point must be emphasised, however, that the corrections for these illusions are very small and that overdoing them is worse than overlooking them.

Free-hand Single Stroke Lettering.—The term "single stroke" means that the width of a single stroke is the width of line used in making the letter, and "free-hand" means that no guiding instruments are employed and the pen is used as in writing. This is the most commonly used lettering, and a knowledge of its construction is very necessary for anyone who makes charts. The ability to letter well can be acquired by anyone who has normal control of his fingers, who will practice faithfully, and observe carefully the shapes of the letters. There is no direct relation between one's lettering and his handwriting. Many good letterers write very poorly.

In lettering, the pen should be held lightly, as in writing, and the strokes drawn with a uniform motion, with a light, even pressure on the paper. The stroke should be complete in itself and should not be sketched in or retraced. The nibs of the pen should not spread. If a heavier line is wanted, a coarser pen should be used. Horizontal guide lines should always be penciled in as an aid in making the tops and bottoms of the letters even

and straight (Fig. 31). Other lines may be ruled in at random to aid in making strokes vertically or at a certain slant (Fig. 31). The letters may be made directly in ink. Penciling is necessary in composition or laying out symmetrical titles.

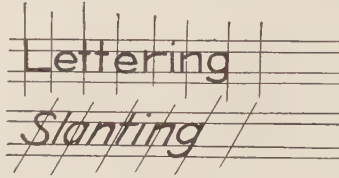


FIG. 31.—Guide lines for lettering.

Vertical strokes are always made downward and horizontal strokes from left to right.

Vertical Single Stroke Capitals.—The forms of these letters must be very carefully observed and should be made exactly as



FIG. 32.—Vertical single stroke capitals. (From "Engineering Drawing" by Thos. E. French.)

shown in Fig. 32. They are all made the same width except M, W, I, and the numeral 1. Cross lines are drawn slightly above the center, except for A and G and the figures 4 and 9,



FIG. 33.—B, E, F, H, J, L, P, R,



FIG. 34.—6, 9, and 0 superimposed, and U superimposed.

which are a little below. The similarity in the proportions of the letters B, E, F, H, J, L, R, and U is brought out by Fig. 33, which shows how they would look if superimposed one upon the

other. The sides of the letter M should be made parallel and not sloping as in an inverted W.. A very important point to observe in making the letters B, D, P, and R is that the tops and bottoms are made mainly of horizontal strokes. Figures are made the same height as capitals and sometimes slightly narrower. The backbones of the 6 and the 9 have the same curve as the cipher. Figure 34 shows these three figures superimposed.

Vertical Lower Case.—The alphabet of vertical lower case letters is made as shown in Fig. 35. These letters are all of the



FIG. 35.—Vertical lower case. (From "Engineering Drawing" by Thos. E. French.)

same width except the seven letters, f, i, j, l, m, t, and w. The bodies are made two-thirds of the height of capitals, with the ascending stems reaching the cap guide line and the descending stems reaching the same distance below (Fig. 36). The circle and parts of it combined with straight lines form the basis of the letters a, b, c, d, e, g, h, n, o, p, q, r, u, and y. These fourteen letters are shown superimposed in Fig. 37.

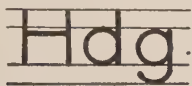


FIG. 36.—Proportions of letters.



FIG. 37.—a, b, c, d, e, g, h, n, o, p, q, r, u, and y superimposed.

Inclined Capitals.—The inclined or slanting alphabet shown in Fig. 38 is easier to make than the vertical, as a slight difference in inclination is not apparent, while a slight tendency from the true vertical is very noticeable. The inclination is generally made from 60 to 70 degrees to the horizontal. Special triangles are made for drawing these slope lines. Particular attention must be given to the inclined A, V, and W, in that a slope line drawn through the point of the letter should bisect the angle made by the two sides of the letter (Fig. 39). The discussion in

the preceding paragraph on vertical capitals applies equally well to the inclined capitals.

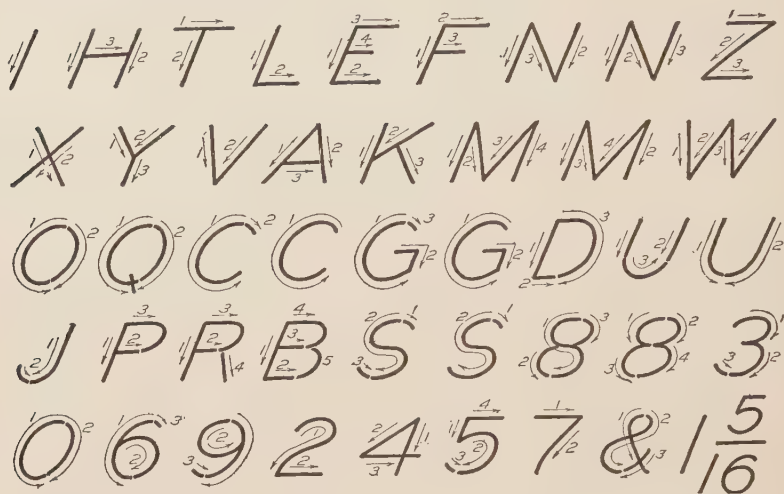


FIG. 38.—Inclined capitals. (From "Engineering Drawing" by Thos. E. French.)



FIG. 39.—Slope of inclined A, V, and W.

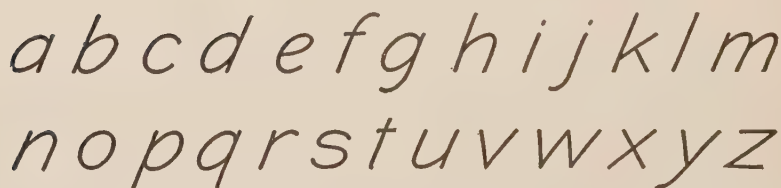


FIG. 40.—Inclined lower case. (From "Engineering Drawing" by Thos. E. French.)

Inclined Lower Case.—The inclined lower case letters (Fig. 40) are founded upon the straight line and the ellipse instead of the circle. In other respects they are proportioned the same as in the vertical system.

Common Errors to Be Avoided. Some common errors to be avoided in both capital and lower case letters are illustrated in Fig. 41.

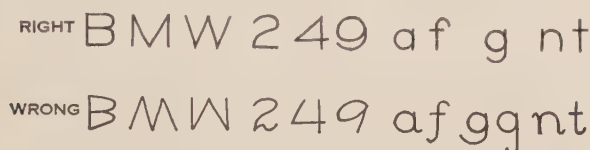


FIG. 41.—Common errors.

Large Commercial Gothic.—The preceding description of letter-making has considered only the free-hand, single stroke method of making the Commercial Gothic style of lettering. For large careful work the letters are outlined and filled in (Fig. 42).

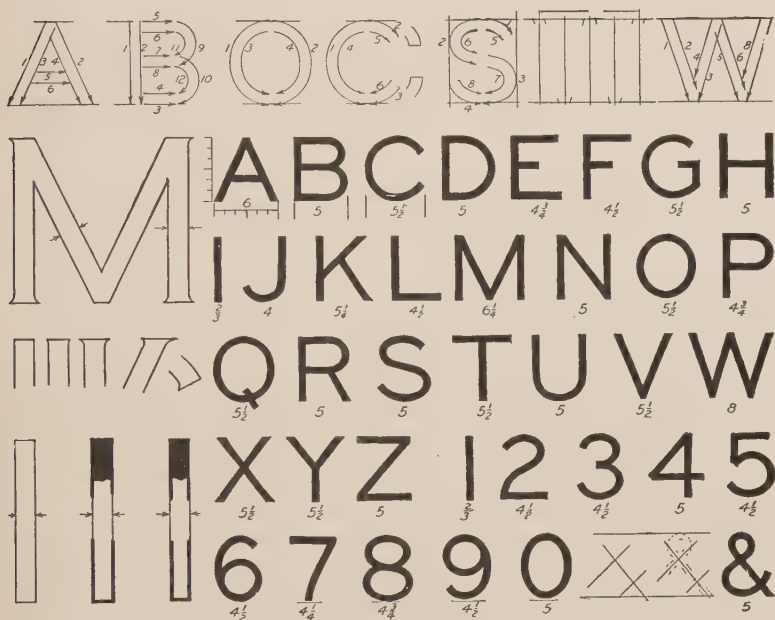


FIG. 42.—Outlined letters for large careful work. (From "Engineering Drawing" by Thos. E. French.)

When inking a penciled outline, the ink should not extend outside of the pencil line or the letter will be made heavier than was intended. For work to be read from a distance, brushes or the methods mentioned on pages 23 and 24 in the description of Payzant and automatic marking pens sometimes are used, but lettering made in these ways will not stand careful inspection.

Handmade Modern Roman.—The Roman is the most familiar of all styles of lettering, as it and its modifications are generally used in present-day printing. It is, however, a difficult lettering to make and can be mastered only by very careful attention to the details of forms and proportions.

Exact rules for the formation of these letters are of little value, as success in making them depends upon the judgment of the draftsman. The two widths of lines, which were originally the upstroke and downstroke of a flexible reed pen, must be arranged in the right order. It is a very bad error to shade a Roman letter wrongly. The serifs at the ends of the strokes are a very important part of the letter as far as appearance is concerned. They should be joined by a short curve and should not be angular. These letters are usually made in outline, either free-hand or with instruments, depending upon the size, and then filled in.¹

Spacing.—Definite rules for spacing letters cannot be laid down because of the varying proportions of the letters. An appearance of uniform spacing is a matter for artistic judgment and cannot be gained mechanically. If letters are spaced at equal distances along the guide lines, there will appear to be more space between letters such as AT or OC than between letters with parallel sides such as HN. To give the appearance of equal spacing, the minimum space should be used in such combinations as AT, AV, LY, or TJ, where part of one letter may even extend over part of another, while the maximum spacing should be used in straight side combinations such as HE, NU, or IR. A common mistake of beginners is making letters too narrow for their height and spacing them too far apart.

Spacing between words varies with the effect desired but should always enable the words to be read easily. In ordinary work the space should appear to be about one to one and one-fourth times the width of the average letter used.

It is very important to allow ample space between lines if the work is to be read easily. This space should be from one-half to one and one-half times the height of the capitals. The crowded appearance of chart lettering is often due to the fact that not enough space has been left between lines.

¹ For further information see French and Meiklejohn, "Essentials of Lettering."

In laying out a symmetrical title for a chart it usually is best to balance it on a vertical center line. The letters and spaces in the title are counted and then sketched in pencil, beginning each line with its middle letter at the center line.

Mechanical Lettering.—The term “mechanical lettering” is intended to include all methods in which the letter is produced, not by being built up each time, as in hand lettering, but by some already produced letter form which may be used over and over again. The term is broader than printing (with type) as it includes such methods as stenciling and the use of cut-out letters.

The Drafting-room Printing Press.—Small hand printing presses are made especially for printing on charts and drawings. These presses do any kind of ordinary lettering work much better and faster than the average draftsman can do it. They are simple in construction and require but little training for efficient operation.

Rubber Stamps.—Rubber stamps can be used for making reasonably good lettering providing they are skilfully used and with the right kind of ink. A uniform impression can be made with a large stamp by using a simple, inexpensive, rubber type printing press. By using the “Stamp-o-graph process,”¹ opaque, smear-proof letters can be stamped on tracing cloth.

Pasting on Printed Titles.—Very good results can be obtained, when making charts that are to be reproduced by a photographic process, by pasting on a title that has been set up and printed by the ordinary printing process. Several titles can be set up and printed together at a small expense. These can be cut apart and mounted perfectly flat with rubber cement (see page 19). If the edges are painted with Chinese white, they will not show when photographed and the letters will appear to be printed directly on the chart. This probably is the best method of all for lettering charts when high-grade cuts, and photographic, or photostatic reproductions are wanted.

Cut-out Letters.—Gummed letters cut out of black or any colored paper are made for setting up titles and are used simply by ruling in guide lines and pasting on in order. A great many grotesque forms of these letters are on the market and care should be taken to select forms that look like printed letters. They may be purchased at any well-supplied stationery store.

¹ Eugene Dietzgen Co., Chicago.

The Typewriter.—This machine may be used successfully in making small charts. There are not, however, enough differences in sizes of type on an ordinary typewriter to give the lettering the right proportions for the various titles, captions, keys, and references usually placed on graphic work. This situation may be improved somewhat by using special machines such as the "Hammond Multiplex" or by using two or more machines with different sizes of type, if they are available. Letters may be typed in various colors without changing the ribbon by using colored carbon papers.

CHAPTER III

PRINCIPLES OF CHART PLANNING

There are several principles, common to all of the different types of charts, which must be considered before work begins on any one in particular. One of the most important of these is to decide definitely upon the purpose of the chart. Also size, proportion, form, title, etc., must be decided upon. To consider these is the object of this chapter.

Purpose of the Chart.—In planning the construction of charts, a cardinal principle to be remembered is that the reader's point of view must always be kept in mind. When there is a question as to which of several methods should be used, that method should be chosen which will enable the reader to grasp the point the most easily and clearly. It often happens that the chart which is easy to make is difficult to read, and that the chart which is easy to read is difficult to make. However, since the chart is made for the use of the reader rather than for the maker, the decision on any point should be in favor of the reader.

When charts are prepared for the use of the skilled executive, and at his request, it is to be presumed that he will make an effort to learn enough of the technical principles of the graphic language to enable him to read his charts effectively. But when the purpose of a chart is to make an appeal to the casual or untrained reader, simple methods, which are already familiar to the person appealed to, must be used. Plain technical graphs will be used by the executive or the board of directors who already have an interest in the subject-matter to be presented. On the other hand, charts for the untrained reader must be in some popularized form such as those in which pictures or cartoons help to bring out the facts represented. Charts will vary all the way from popularized cartoons to graphs of the scientific type, depending upon the prospective reader's training. In popular periodicals the tendency naturally is toward the use of simple diagrams, while in technical publications there is an advantage in using the customary technical forms.

Size and Proportions of the Chart.—The size of a chart should be determined by considering its convenience in handling and ease of reading. It is often best to have several small charts instead of a large one, especially if this large chart has to be folded. A folded chart is always hard to handle and is easily damaged. There is a tendency to make charts larger than necessary but, as a general rule, it is best to use the smallest size that can be read conveniently. The larger the number of charts that can be placed upon one page, without making them so small that they are hard to read, the greater will be the convenience in using the charts.

When it is desired to make an exceptionally long chart, for instance a time series plotted by months over a period of ten

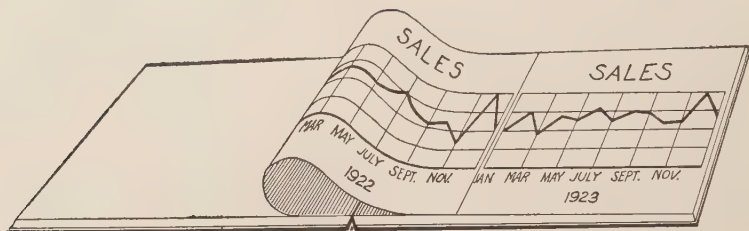


FIG. 43.—Consecutive pages forming a continuous chart.

years, it may be made more convenient to handle by breaking it up into, say ten charts, and binding together in book form. The consecutive pages form a continuous chart across the break between pages when used as illustrated in Fig. 43.

When preparing a chart, it is usually well to outline it first as a rough draft in order that the best proportions can be determined for the final chart.

When making a large chart, it is advisable to lay it out in detail in small form and then enlarge to the desired size. For instance, the wall chart of Fig. 44 was first laid out $7\frac{1}{2}$ by 12 inches and then enlarged to $3\frac{3}{4}$ by 6 feet. All dimensions, widths of lines, lettering, etc. were enlarged exactly six times (linear). The small chart should, of course, be carefully checked for accuracy before enlarging.

Charts may be enlarged by a draftsman by hand, or by photographic processes. If enlarged by a draftsman, they are usually drawn in pencil on heavy paper and traced on tracing cloth or paper from which a Van Dyke and brown- or blue-line prints are made. If enlarged by a photographic process, a small

plate is made with a camera. From this an enlargement of the desired size is made on a special thin paper. Then from this a Van Dyke can be printed and brown- or blue-line prints made as with the hand-made enlargement mentioned above. (For a description of brown- and blue-line printing see Chap. X.)

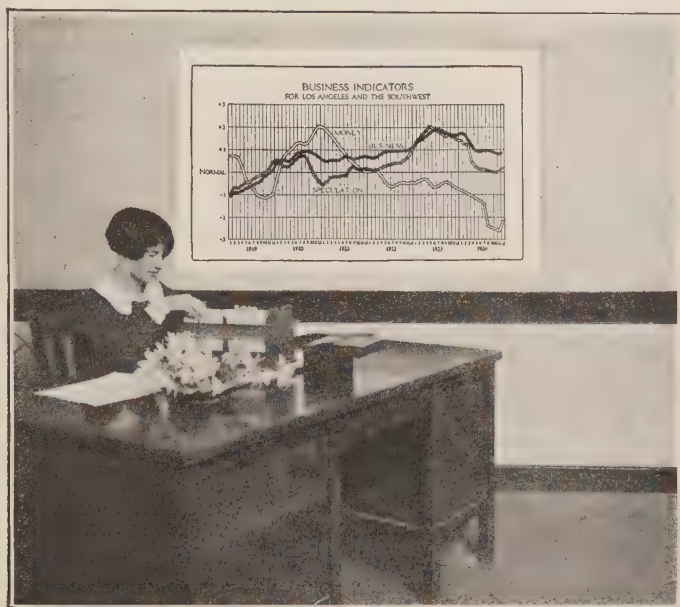


FIG. 44.—Wall chart. This chart was enlarged in direct proportion from a chart 12 inches in length.

Printed Forms for Charting.—Various kinds of ready-ruled coordinate paper can be used in chart making, but for many business graphs it is easier to rule a special form to fit the problem than it is to fit the problem to a ready-ruled form. Coordinate paper, however, can be obtained in special forms for business charting (Fig. 45), which are convenient for rough work but are rarely suitable for fine work such as that prepared for publication.

In most cases extreme accuracy in coordinate paper is not required; also in most cases the finished chart should not show more coordinate lines than are necessary for the purpose of reading to the required degree of accuracy. It very rarely happens that standard coordinate paper is so ruled that it has the desired divisions or presents a good appearance when finished. It is, therefore, frequently desirable to make up zinc plates or to use

the "offset" printing process in order to secure just the kind of coordinate paper required. In some cases permanent titles, etc., can be printed on the original paper. If the scales are properly laid out, accurate plotting can be done between the guide lines by using the right decimal scales, by using the scale as illustrated in Fig. 5, or by making special scales out of thin strips of cellu-

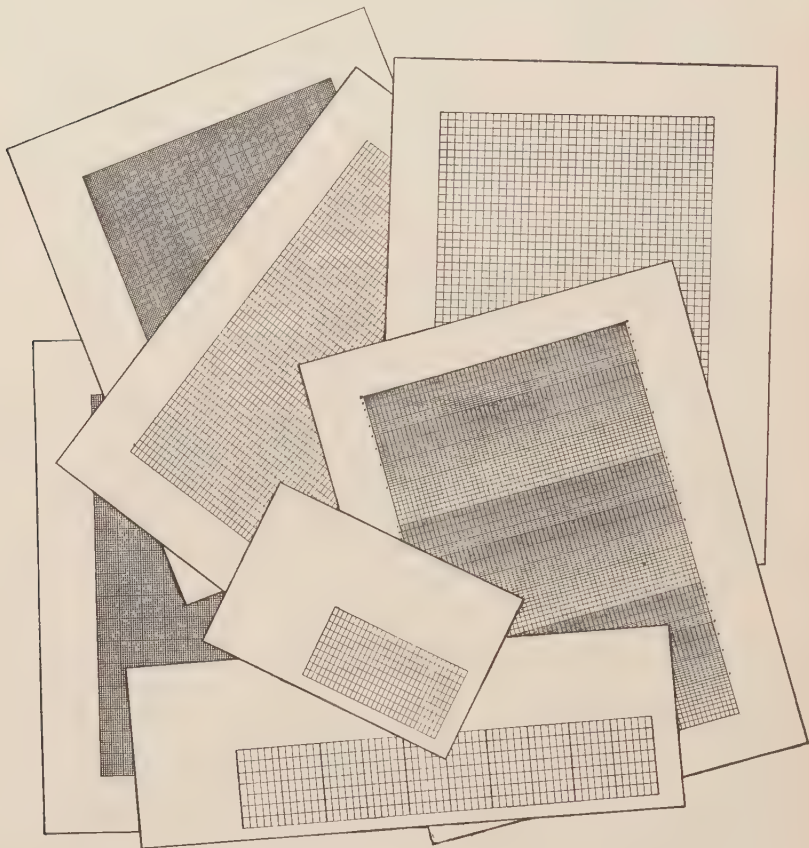


FIG. 45.—Coordinate paper. (*Courtesy of Educational Exhibition Co.*)

loid, marking the divisions with a knife-edge and filling the grooves with black ink.

Standardization of Charts.—A considerable amount of time and labor can be saved by standardizing graphic forms where possible. By using charts of the same size and with scale intervals of the same values, comparisons can be made between

different charts as illustrated in Figs. 46 and 47. With the charts arranged vertically, one above the other, a direct comparison of seasonal changes can be made, and when arranged horizontally, or end to end, it is easy to see the changes which have occurred in the curve throughout the period as a whole.

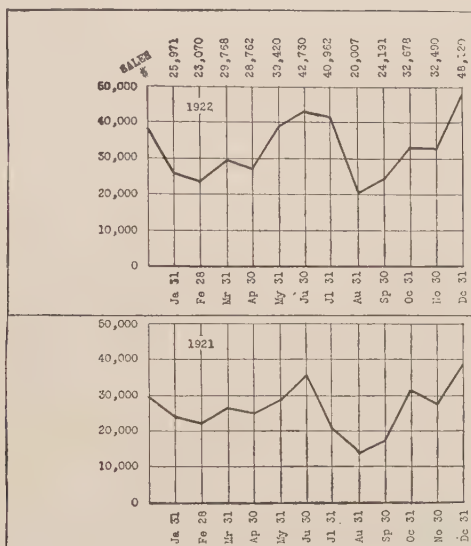


FIG. 46.—Cards placed one above the other for comparing seasonal variations.

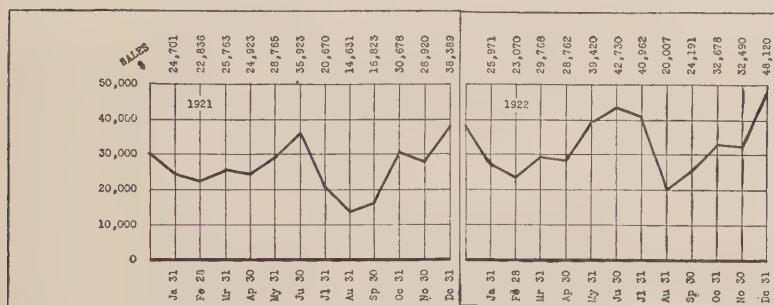


FIG. 47.—Cards placed end to end to form a continuous chart.

Degree of Accuracy in Charting.—In charting business statistics a reasonable degree of accuracy is, of course, necessary, but the problem differs from that of plotting technical data where measurements are made from the charts with finely graduated instruments. If the chart is accurate to the degree that visual

comparisons will not be affected, it is accurate enough for most practical purposes. Care should be taken however, to have charts accurate to this degree. Carelessness in plotting or checking will cause charts to be very misleading.

It is a good plan to include the data in tabular form along with a chart, either as part of the chart or as a separate table. This is especially valuable to readers who wish to make further use of the subject-matter presented.

The source of the data presented should be given on the chart in all cases where it is not evident from the title or the nature of the data. It is important for the reader to know the source for it helps him to judge the reliability of the chart. He may also check the chart for accuracy and, knowing the source, he may secure further data in connection with the subject represented.

Position and Size of the Title.—Titles have been placed on various parts of business charts in the past but at present the

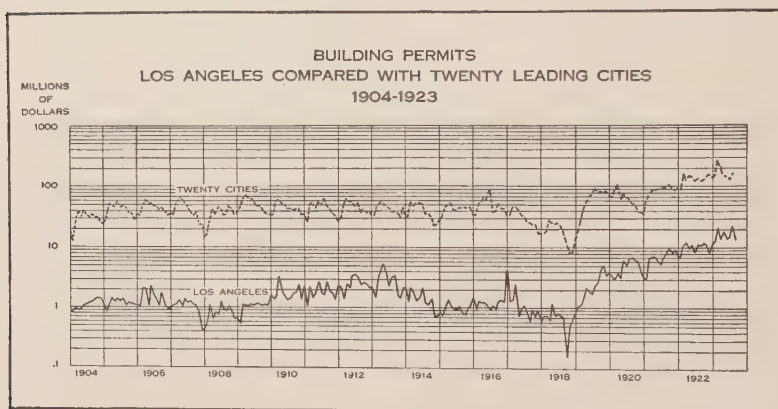


FIG. 48.—Correct position for a chart title.

weight of authority agrees in placing the title at the top of the chart as illustrated in Fig. 48. This is convenient and natural from the point of view of the business man since his bills, tables, books, accounts, schedules, and other forms are commonly labeled at the top.

No general rule can be laid down for determining the size of the title lettering. Only skill and experience can best decide the most appropriate size for each particular problem. However, the title lettering should be the largest and most prominent of

any on the chart, and by studying other charts one can soon become efficient in determining the sizes for the problems in hand.

Importance of a Clear Title.—One of the most important parts of a business chart is its title. The title should give the reader the subject of the chart clearly and completely. It is often given very little attention by the draftsman, and though the chart may be very clear in its other parts, it may not be read at all without a clear and complete title.

If possible, the title should be short, but it may be long if necessary for the sake of clearness. It may consist of a major title with a subtitle which adds further explanation to the main subject. It is sometimes difficult to avoid technical words but here, as in the rest of the chart, the wording must be chosen with

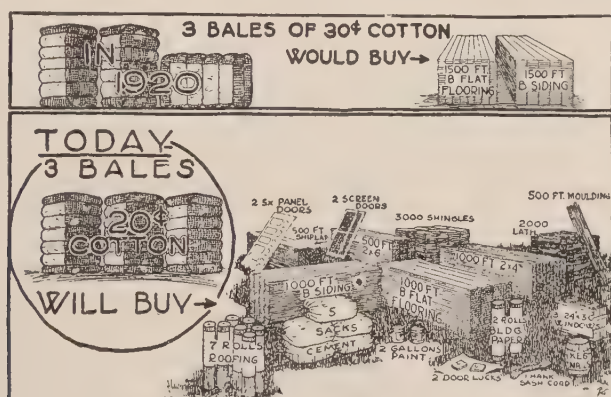


Fig. 49.—Pictorial chart for popular presentation. (Courtesy of Retail Lumberman.)

the point of view of the reader in mind. Superfluous wording should not be used as, for instance, in the title "Chart Showing Curve of Jones Brothers' Sales for 1922." Here the first four words are unnecessary, as any one would recognize a chart showing a curve. A better title would be simply "Jones Brothers' Sales for 1922."

Use of Color.—Charts in which colors are used to bring out differences or contrasts are much more effective than those in which only black is used. Colors, however, are limited in their use to work in which their cost is not prohibitive. When charts are not to be reproduced, it is no more expensive to use colors than black ink alone. But if the chart is to be reproduced in a

printed report or magazine, for instance, it will usually be impossible to use colors because of the high cost of color reproduction.

Pictorial Charts.—When prepared for popular use, charts are often much more effective if made in a pictorial form, as shown in

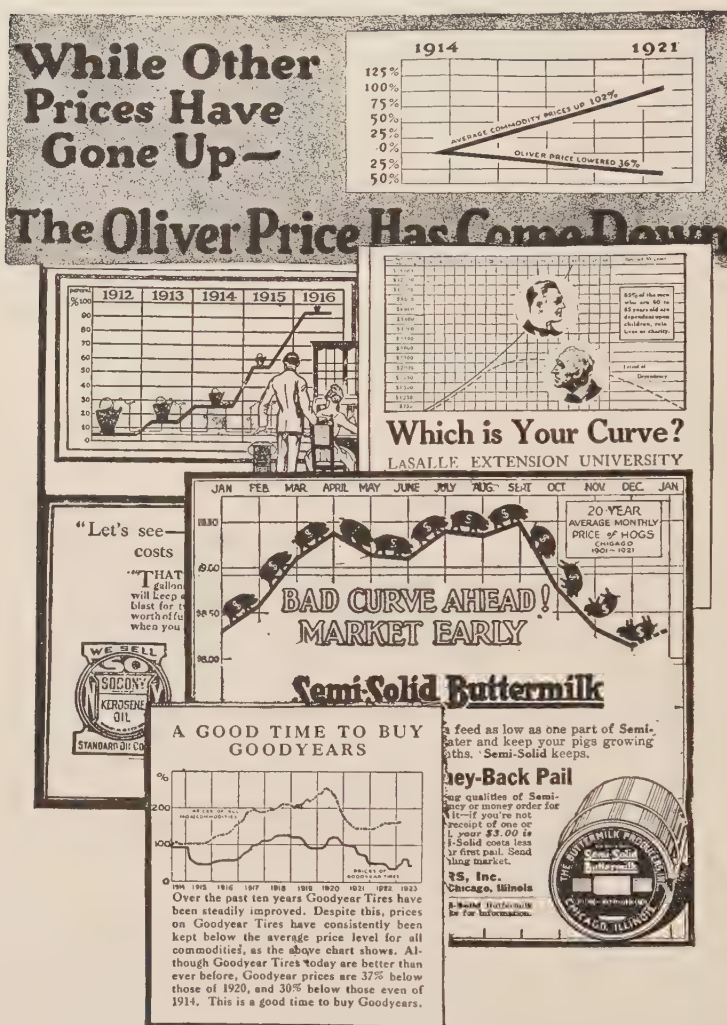


Fig. 50.—Popular adaptations of curves in magazine advertising.

Fig. 49, or with pictorial additions and ornamentations which help to direct attention to the subject-matter, as illustrated in Fig. 50. These methods of popularizing may be applied to most

of the more technical forms of business charts. The construction and use of the pictorial form are discussed and illustrated at length in the chapter on simple comparisons of size (Chap. IV).

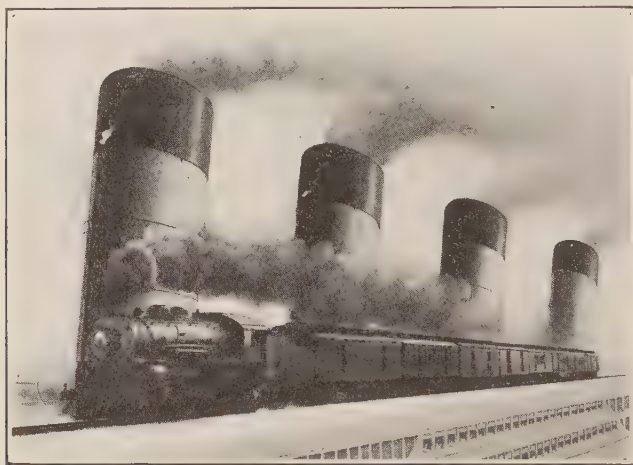
General Appearance.—Confidence in a chart is much greater if it is neat and accurate rather than slovenly in appearance. Skill and neatness come with practice and experience. The chart must be well proportioned; it must not be crowded; and ample margins must be left. Usually a border line will improve the appearance of a chart, and it is of utmost importance in securing an appearance of high quality to have the title and all other lettering done neatly and carefully.

It should be noted that in a chart like Fig. 48 the title should be centered between the vertical border lines—not over the center of the space covered by the coordinate lines. Room must be allowed for the scale, and hence the center of the coordinate ruling will be a little to the right of the center of the area within the border lines. In order to have the right and left margins appear to be equal, less space should be allowed in the margin on the left between the scale and the border line than between the last ordinate and the border line on the right. The reason for this is that the space between two solid lines appears to be less than an equal space between a solid line and a scale with its open spaces.

CHAPTER IV

SIMPLE COMPARISONS OF SIZE

It is the purpose of this chapter to describe the graphic methods of comparing magnitudes where only the sizes vary. There is no consideration of time variations, geographic distributions, or other variables. For instance, we might compare the populations of New York and Chicago, or the heights of buildings in New



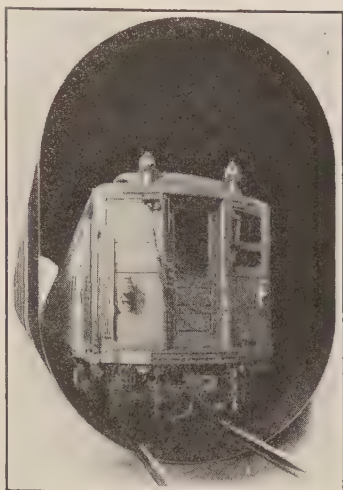
An eight-car train, with locomotive and tender, could be placed upon the upper deck of the "Aquitania" and reach less than half the ship's length.

FIG. 51.—Pictorial comparison of size. (*Courtesy of the Cunard Steamship Co.*)

York, or the sales made by different salesmen, or the profits made on clothing with those made on shoes.

Whatever the comparison, the object is to show relative sizes. The object may be to indicate the relation of one size to another where both are known, or it may be to give a conception of an unknown size by comparing it with a known size. For instance, most persons are familiar with the size of a passenger train. In order to impress the greatness of the size of a modern steamship upon the readers of a popular magazine, the graphic comparisons shown in Figs. 51 and 52 were made. Such comparisons, however,

are very general, and can be used only when the actual objects can be compared. In most cases of graphic presentation of sizes, the comparisons are made by using certain geometric forms which vary in their sizes in proportion to the different magnitudes represented.



A New York subway train could pass through one of the funnels of a great ocean liner with room to spare.

FIG. 52.—Pictorial comparison of size. (*Courtesy of the Cunard Steamship Co.*)

Choice of Geometric Forms for Simple Comparisons of Aggregates.—When one is about to undertake the graphic comparison of simple sizes, the first thing to be decided upon is the form to be used. The two fundamental requirements here are that it must be easy to understand and it must give a correct impression.

It is easier to compare the sizes of forms which vary in one dimension than those which vary in two or three dimensions. It would be very difficult for the average person to compare the sizes of New York and Chicago by looking at the cubes in Fig. 53A because they are drawn on a volume basis and vary in three dimensions. Also it would be difficult to compare these cities by use of the squares shown in Fig. 53B which are drawn on an area basis. In Fig. 53C the lines or bars vary only in one dimension, and it is easy to see that New York is about twice as large as Chicago. It is this form that is most commonly used in making graphic comparisons of size, as it is much easier to compare

the lengths of bars than it is to compare areas which vary as the square of their sides, or volumes which vary as the cubes of their edges.

Circles are used occasionally to represent sizes, but it is difficult to compare them and they should not be used when it is possible to use bars. When it is necessary to use circles, as, for

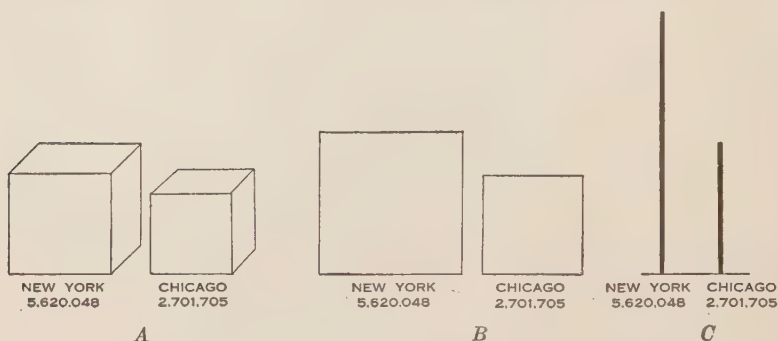


FIG. 53.—Use of different geometric forms in comparing sizes. *A*, Volumes are very difficult to compare. *B*, Areas are difficult to compare. *C*, Lines (or bars) are easy to compare.

instance, on certain kinds of statistical maps, they should be drawn on an area basis rather than on a diameter basis. Sometimes, however, circles are drawn on a diameter basis, and often it is not known which method is used. Figure 54*A* compares New York and Chicago by means of circles drawn with areas

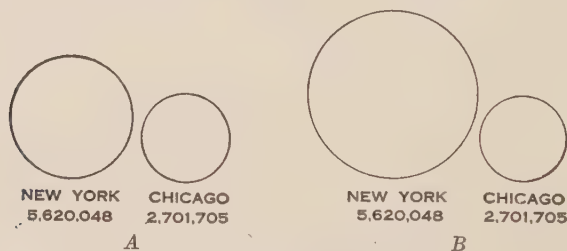


FIG. 54.—*A*, Circles with areas proportional to numbers. *B*, Circles with diameters proportional to numbers.

proportional to population, and Fig. 54*B* compares them with diameters proportional to population. Figure 54*A* gives the impression that New York is somewhat less than twice the size of Chicago while Fig. 54*B* gives the impression that New York is much more than twice as large.

Pictorial Forms.—The principles just discussed in connection with simple lines, areas, and volumes must be kept in mind when using pictorial forms.

It is especially difficult to compare irregular areas such as those shown in Fig. 55. Though the method is quite effective in

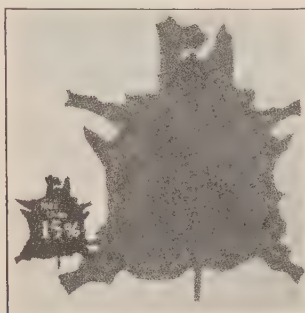


FIG. 55.—Pictorial comparison of size on an area basis. (*Courtesy of Swift and Co.*)

showing the readers of a popular magazine that Swift's tanneries handle but a small part of the total, a more accurate picture would be given by using a chart like Fig. 56. Swift does not tan smaller hides but a smaller number of hides. In making a popular appeal, the use of the illustrations of hides in both Figs. 55 and

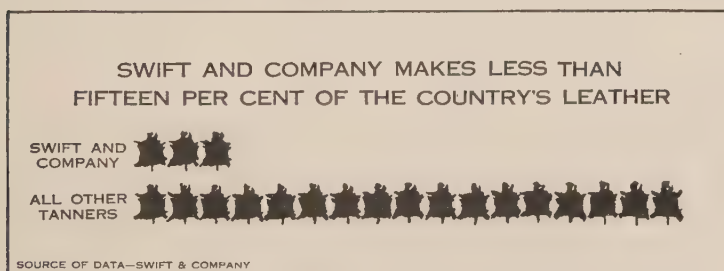


FIG. 56.—Pictorial comparison of size on a line basis. Easier to read than Fig. 55.

56 makes the chart much more effective than would the use of ordinary bars. It is still more difficult to make the comparison in Fig. 57, since the volumes must be considered. Here the attempt to make a striking appeal has been very successful, which probably is the principal object of the chart, but it is not easy to make an accurate comparison of size.

The purpose of Fig. 58 is to compare sizes of cities. It is supposed to show, for instance, that Buenos Aires is approximately five times as large as Washington while, as a matter of fact, the effect is that Buenos Aires is much more than five times

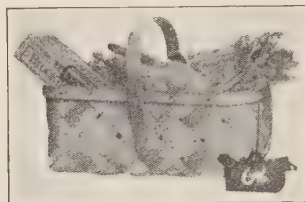


FIG. 57.—A striking appeal, but difficult to compare sizes. (Courtesy of Swift and Co.)

as large. The chart is so drawn that the heights of the figures are proportional to the numbers represented. Charts of this kind are quite common and usually give false impressions, as the reader



FIG. 58.—A poor method of comparing the populations of the capital cities of the Americas. Heights of persons are proportional to population represented. The reader unconsciously makes the comparison on a volume basis.

compares the figures on a volume basis which greatly exaggerates the actual differences in the numbers represented. If it is desired to use persons in popular presentations of this type, it is far better to represent the numbers, not by the heights of persons, but by

rows of persons evenly spaced with the lengths of the rows proportional to the numbers represented. Buenos Aires does not have larger people than Washington but a larger number of

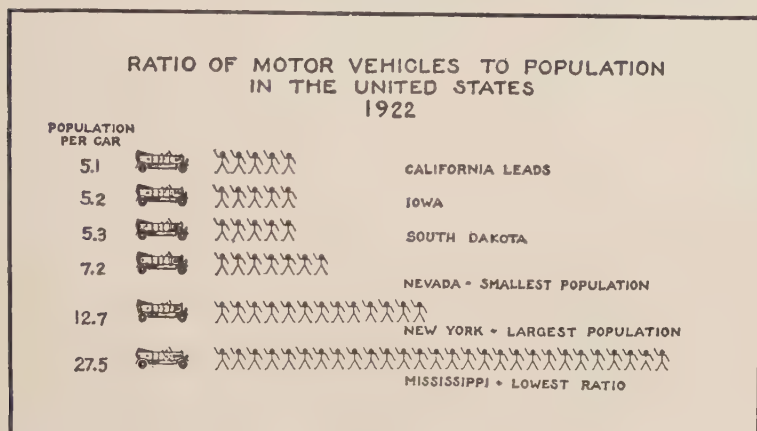


FIG. 59.—Better than Fig. 58. Comparisons are made on a line basis.
(Adapted from the National Automobile Chamber of Commerce.)

persons. This principle is brought out by Fig. 59 which shows the number of persons per car for given states. The figures are equally spaced with the lengths of the rows exactly propor-

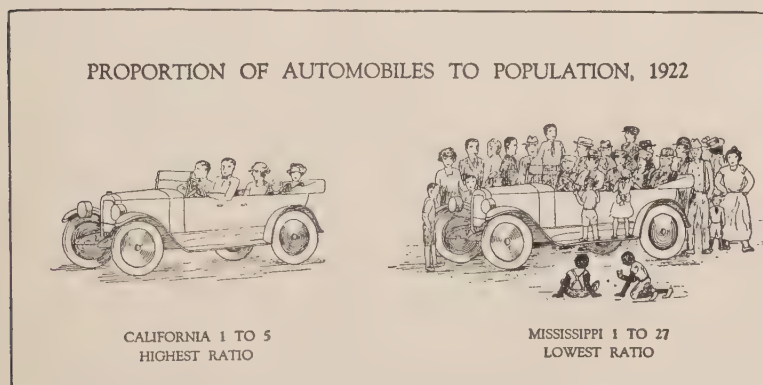


FIG. 60.—Good for certain types of popular presentation.

tional to the numbers of persons represented. Another method of making a popular presentation of similar automobile figures is illustrated by Fig. 60. When it is desirable to use figures of persons to express numbers, methods such as shown in

Figs. 59 and 60 should be used, for they have all of the advantages of Fig. 58 and do not have the serious fault of leaving a very inaccurate impression.

There are many figures that can be used in popular presentations, which may be made to vary in only one dimension without

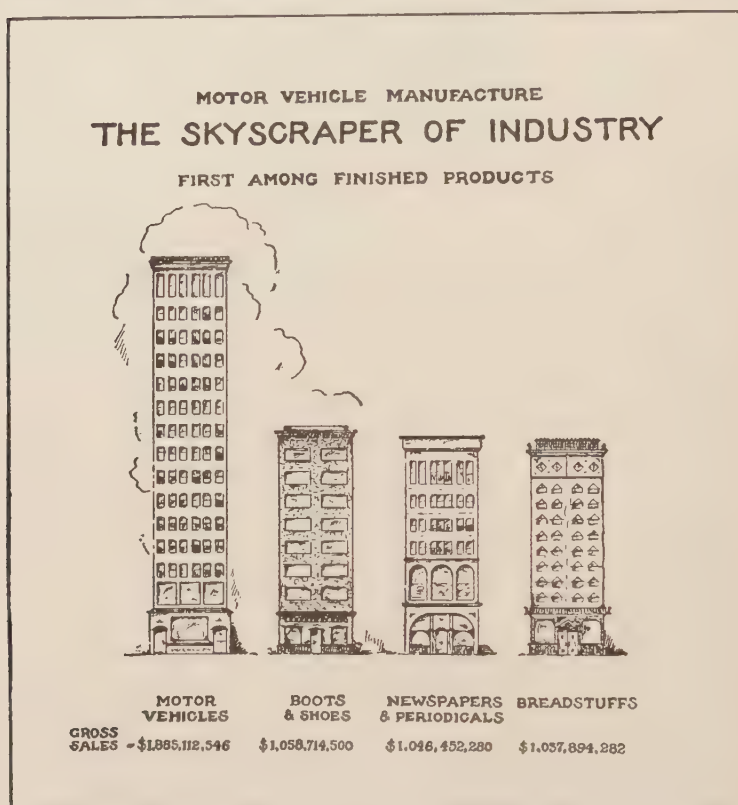


FIG. 61.—Good pictorial presentation (line basis). (*Courtesy of the National Automobile Chamber of Commerce.*)

throwing them out of proportion, as would be the case in varying only the heights of persons. This is illustrated by Fig. 61 in which height is the only dimension that changes. It is very easy to see from this chart that automobile sales are nearly twice that of each of the other three products represented. Note that the title "The Skyscraper of Industry" makes the use of buildings instead of bars particularly significant. This chart would be improved by using the same style of front for all of the build-

ings, since the only difference to be represented is that of size measured in terms of dollars.

There is quite a common use of spectacular designs, such as Fig. 62, which mean nothing in particular, as it is practically

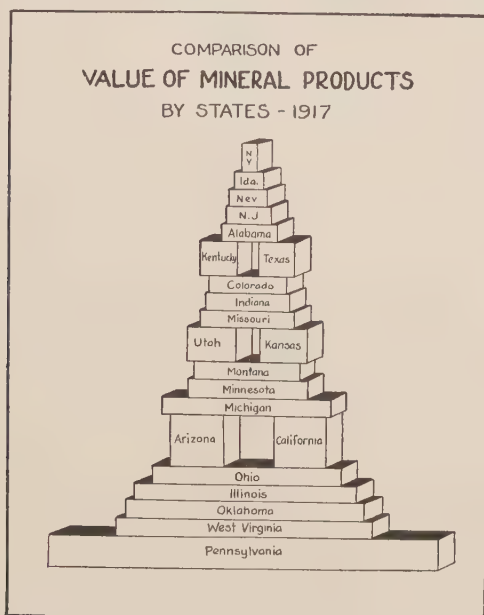


FIG. 62.—Very poor presentation.

impossible to interpret them. In this instance there is little, if any, consideration on the part of the illustrator for an effective comparison of sizes, as the chief aim is to build up a striking architectural form.

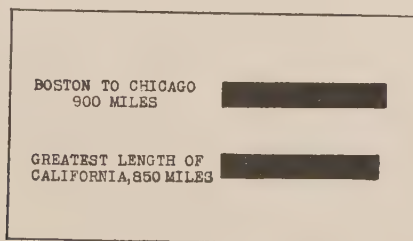


FIG. 63.—Simple horizontal bar chart.

Bar Charts.—The most common method of graphically representing comparisons of size is by one form or another of the “bar chart.” There are several variations of this form of chart

and they have been given many fanciful names, but all are made up of vertical or horizontal bars so drawn that their lengths are proportional to the sizes represented.

For simple comparisons, charts like Figs. 63 and 64 serve the purpose, but when presenting several items for careful analysis, the chart must include certain details such as a properly constructed scale, explanation of units used, table of data, and authority. These must be known in order to read or analyze the chart carefully.

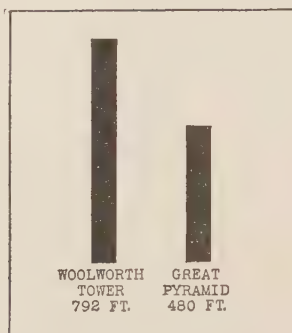


FIG. 64.—Simple vertical bar chart.

Vertical and Horizontal Bars.—There is no particular rule for choosing between vertical and horizontal bars except that those should be used which will enable the reader to understand the chart most readily.¹ In cases where the things represented vary naturally in either a horizontal or vertical direction, the bars usually are made to vary in the same direction. For instance, the bars in a comparison of the distance from Boston to Chicago with the greatest length of California would naturally be made in the horizontal direction (Fig. 63), while those in a comparison of the height of the Woolworth Building with that of the Great Pyramid would be made to extend vertically (Fig. 64). Convenience in reading the lettering is another point that must be considered when choosing between horizontal and vertical bars. If horizontal bars had been used in Fig. 65, the designations of the bars could have been lettered horizontally, which would have made the chart much easier for the average business man to read (compare with Fig. 66). For this reason many statisticians prefer to use the horizontal bar chart exclusively when making simple comparisons of size.

¹ This statement applies only to simple comparisons of size. In showing time series only vertical bars should be used, see p. 84.

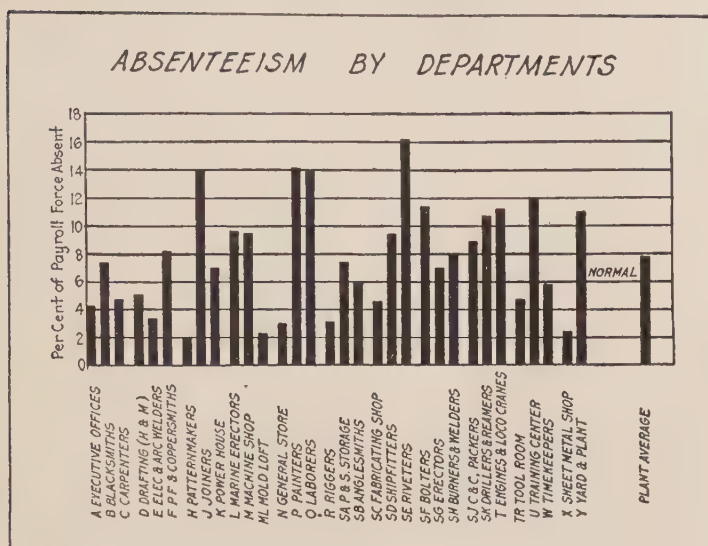


Fig. 65.—Compare with Fig. 66 for ease of reading lettering. (Courtesy of Industrial Management.)

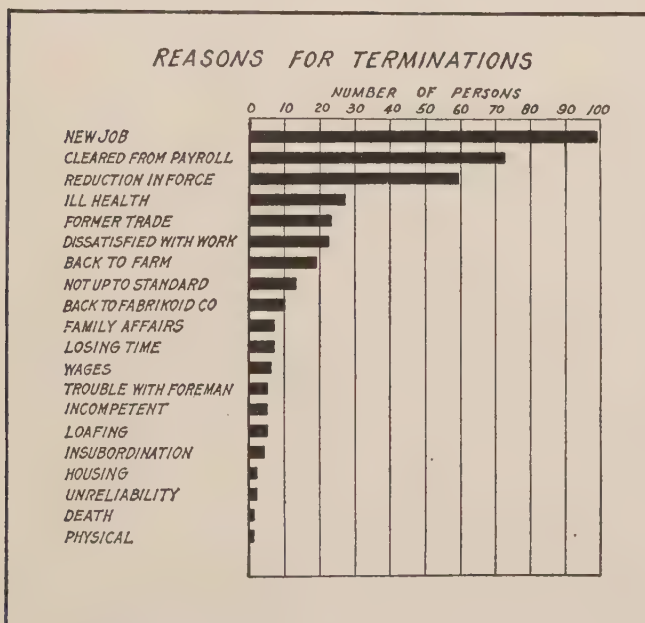


Fig. 66.—Easier for the business man to read than Fig. 65. (Courtesy of Industrial Management.)

Bar Chart Scales.—Except in cases of rough or popular comparisons, it is important to include a scale on a bar chart. A scale enables one to read and check for accuracy much more easily than when it is not given. It is a fundamental rule in making bar charts that the scale begin with zero (Fig. 67). This rule is sometimes broken and the values are inaccurately represented as in Fig. 68. Note that copper is about four times as large as zinc when correctly shown (Fig. 67). Figure 68 gives the incorrect impression that copper is about thirteen times as large as zinc.

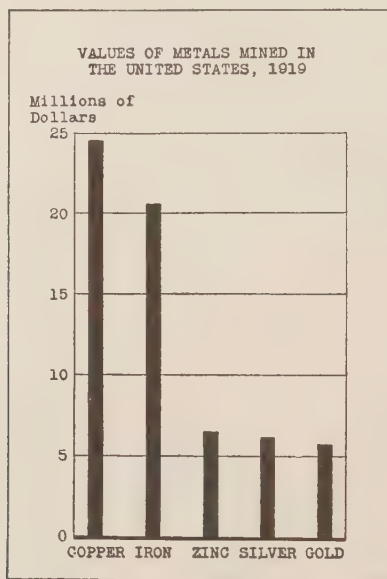


FIG. 67.—Correct use of bar chart scale. The scale should begin with zero

Scales should be laid out in round numbers of convenient intervals. A keen sense of form, appearance, and purpose is the best guide in scale making. These will be gained with experience. The units should be so selected that the eye can read them easily and carry them across the chart with the aid of guide lines (Fig. 67). The zero lines of all charts should be made heavy so they will be sharply distinguished from the other lighter guide lines. Scales such as 0, 3, 6, 9, or 0, 7, 14 should not be used as they are hard to read. It is much easier to read a scale of units, twos, fives, or multiples of ten, or halves and quarters of a hundred, as twenty-five, fifty, seventy-five, etc. Too many numbers

in the scale or too many guide lines in the background are confusing, and hinder rather than aid the reading of the chart (Fig.

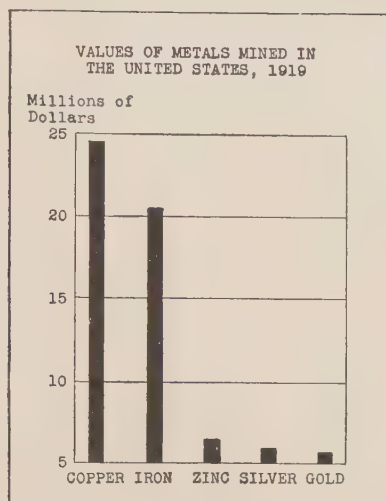


FIG. 68.—Very bad practice. The scale does not begin with zero, hence the lengths of the bars do not represent the sizes accurately.

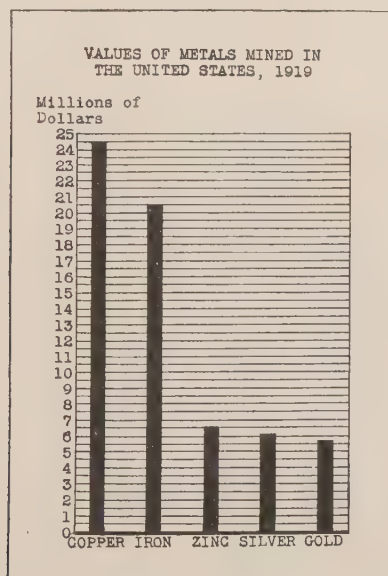


FIG. 69.—Too many scale guide lines across the chart. Compare with Fig. 67.

69). It must be kept in mind that the purpose of graphic methods in business statistics is to present a picture of figures,

and that it is not to make charts from which actual values are to be measured with finely graduated scales or instruments. The aim should be to construct the chart in such a way that it can be understood simply by visual inspection.

The commonly accepted position for the scale of a vertical bar chart is on the left and outside the background of guide lines. Each number should be exactly opposite the line which

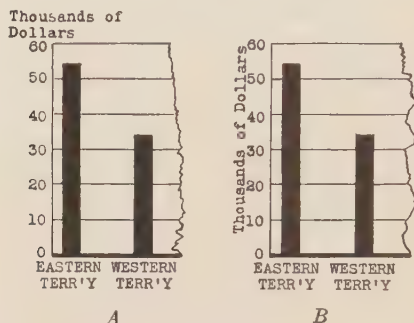


FIG. 70.—Position of scale unit designations on vertical bar charts. *A*, Best practice for business charts. *B*, Common engineering practice.

it represents (Fig. 67). If desirable to have a scale on the right of the graph, one may be placed there in addition to the one on the left, but the left scale should in no case be omitted. The units represented by the scale should be indicated clearly by a caption lettered horizontally at the top of the column of figures. This is preferable to the method of lettering vertically along the

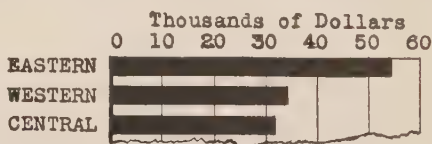


FIG. 71.—Designating horizontal bar chart scales.

side of the column, as the business reader, who often is not adept at reading engineering drawings, does not have to turn his head or the paper (Fig. 70). Expressions such as "1 inch = \$100" are not used in statistical charting. The scale on the graph is much more convenient, and gives all the aid necessary for measuring the different bars.

The scale on horizontal bar charts is commonly placed along the top just above the bars (Fig. 71). It may be placed along the

bottom also if it will add anything to the clearness of the chart. The scale units of Fig. 71 are thousands of dollars, and this fact should be indicated as shown by the designation just above the scale. If there is a scale below the chart, the designation of units would be shown, of course, just underneath the scale.

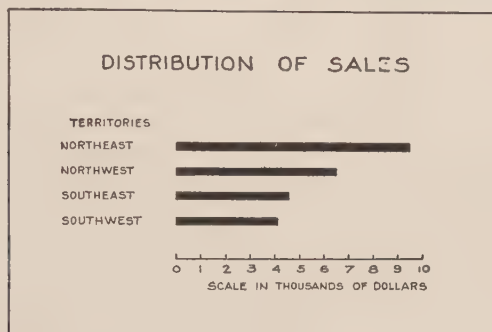


FIG. 72.—A method of including a scale when no background of guide lines is used

Figure 72 illustrates a good method of including a scale when no background of guide lines is used.

Guide Lines to Aid in Reading Scale or Comparing Bars.—

When making charts for careful reading, it usually is best to include a background of guide lines which assist in comparing the bars or reading from the scale (Fig. 73). Judgment must be

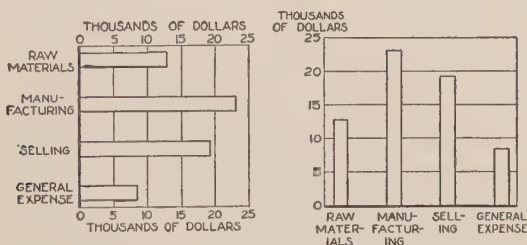


FIG. 73.—Guide lines for reading the scale should not be drawn through outlined shaded or colored bars.

used in selecting the number of lines to be included on the finished chart. Too many lines are as bad as too few (compare Figs. 67 and 69). These lines should not be prominent and should appear to be behind the bars. Thus they should be drawn between, not through, the bars (Fig. 73). This point is not so important, of course, in the case of solid black bars as when shaded, colored, or outlined bars are used.

Designating Bars.—Designations explaining what vertical bars represent usually are placed underneath the bars as shown in Fig. 74. It is always a good plan to present the figures represented along with the chart, and in cases of this kind they may be placed underneath the designations, as in Fig. 74. This position is convenient for recording and reading, and in no way interferes with the effect of the chart. It is not good practice to place figures at the upper ends of vertical bars, as the effect is to add to their lengths, thereby giving a false impression of the com-

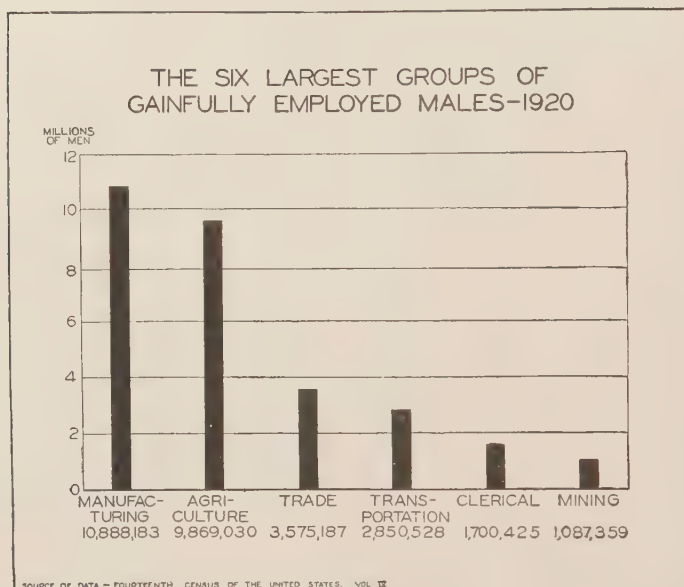


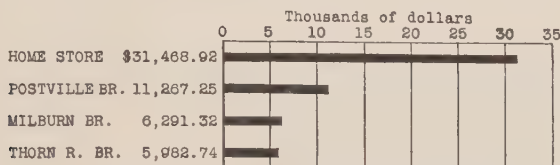
Fig. 74.—Designating bars and tabulating data on vertical bar charts.

parison. When no scale is used and only a few bars are shown, the designations are sometimes placed within the bars, and for popular presentation this is quite effective but it has no advantage over the position used in Fig. 74.

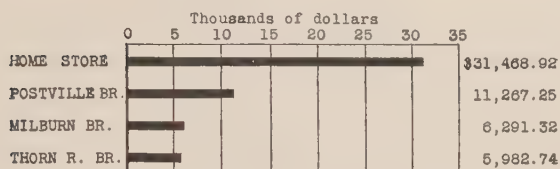
Figure 74 also illustrates the point that, in designating bars, words should be divided if they are too long for the spaces allowed. They should also be lined up with reference to the zero line as shown.

Designations explaining what is represented by horizontal bars are placed on the left as illustrated by Fig. 75A and B.

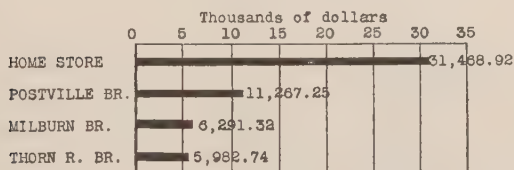
The values represented may be tabulated as shown in Fig. 75*A* and *B*. The arrangement of Fig. 75*A* is preferred, as it is easier to read, especially if there is a great difference in the lengths of the bars. Numbers should not be placed at the ends of horizontal bars (Fig. 75*C*), as the effect is that the lengths of these numbers are added to the lengths of the bars, which gives a wrong impression of the relative values. A still more objectionable plan is to block out designations and data within the bars



A.—A very good method of designating bars and tabulating figures



B.—A good method of designating bars and tabulating figures



C.—A bad method of tabulating figures, as it has the effect of lengthening the bars

FIG. 75.—Designating bars and tabulating data on horizontal bar charts.

as in Fig. 76. This applies to both vertical and horizontal bars. In such cases there is danger that the reader will unconsciously compare the solid parts at the right of the narrow column of figures instead of the entire lengths. In Fig. 76 one cannot tell what is represented by the columns of figures, since no captions or other explanations are given. Charts always should be complete in such details, leaving nothing which might be unintelligible or falsely read to be explained in an accompanying text. Note also in Fig. 76 that the lettering at the extreme left and the

numbers of the scale read from top to bottom, while they should read from bottom to top if they are to be placed vertically. It would be better to turn the scale numbers to read horizontally.

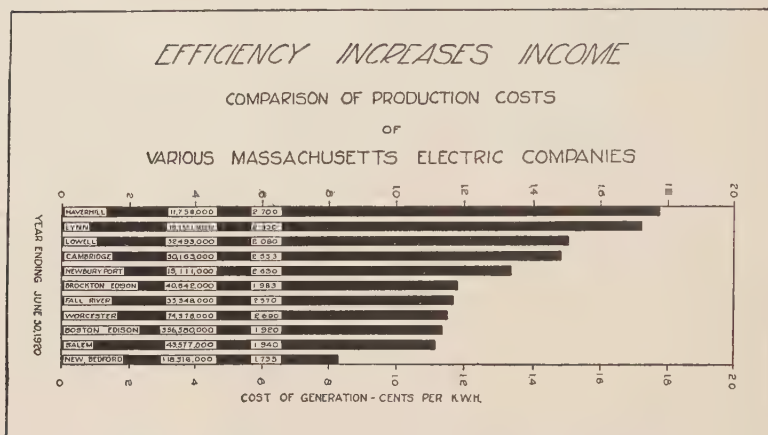


FIG. 76.—Bad practice. Blocking-in has the effect of breaking the bars. (Also lettered incorrectly and lacks explanation of units.)

COMPARISON OF CANALS BY NET TONS OF TRAFFIC, 1919

Sault Ste. Marie 68,235,542
Suez 9,251,601
Panama 6,181,575

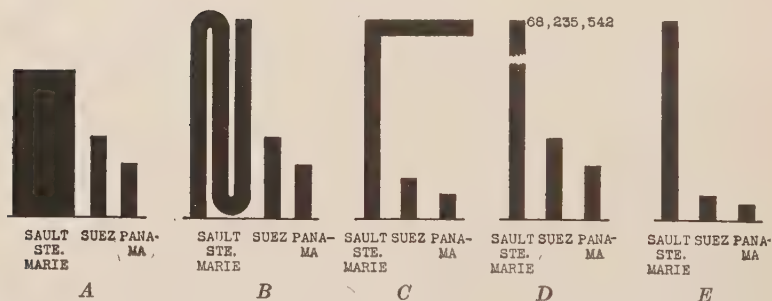


FIG. 77.—Right and wrong practices. A, Wrong. The first bar is four times as wide as the others. Lengths only should vary. B, Wrong. Very hard to compare lengths. C, Wrong. Difficult to compare lengths. D, Wrong. Correct visual comparison is impossible because part of the bar is not shown. E, Right. (A scale and background of guide lines would further improve this chart.)

In this particular case it is possible that the chart was made as a vertical bar chart and turned to a horizontal position by the party who later used it.

Common Errors.—Figure 77 illustrates various bad practices which are quite commonly used by novices in graphic work. There is such a wide difference in the values, that in order to make Panama show up well Sault Ste. Marie would be a very long bar. In such cases, the aim of showing the true relation is often overshadowed by the desire to show the small item. Very small numbers are practically insignificant when compared with large numbers. That is, \$100, for instance, is practically negli-

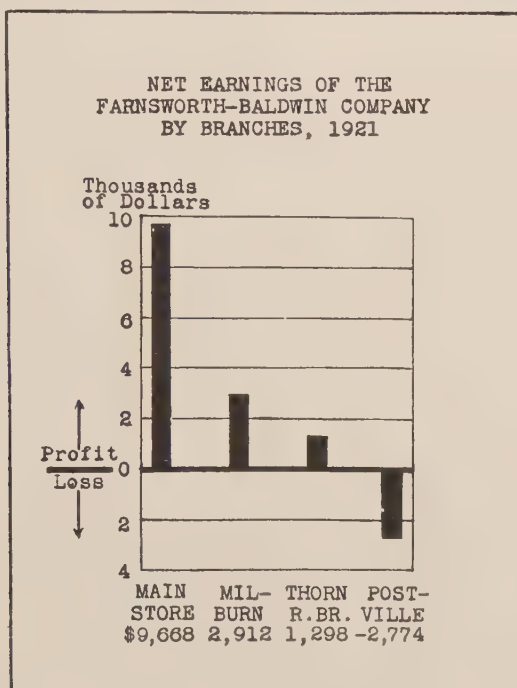


FIG. 78.—Showing loss and gain with vertical bars.

gible when compared with \$1,000,000 and should be shown in that light. All of the shapes and methods illustrated in Fig. 77 were taken from charts published in reports and magazines.

Showing Decreases or Losses along with Increases or Gains.—

So far the discussion of bar charts has considered only positive numbers, which are represented by bars extending vertically above the zero line and horizontally to the right of the zero line. When a decrease or a loss is represented along with increases or profits, it is commonly shown as in Fig. 78 or as in part A of

Fig. 79; that is, vertical bars extend downward from the zero line, or horizontal bars extend to the left from the zero line. Part *B* of Fig. 79 illustrates another method of presenting the percentage change shown in part *A*. In both parts of Fig. 79, the problem is to compare the territories with reference to the change

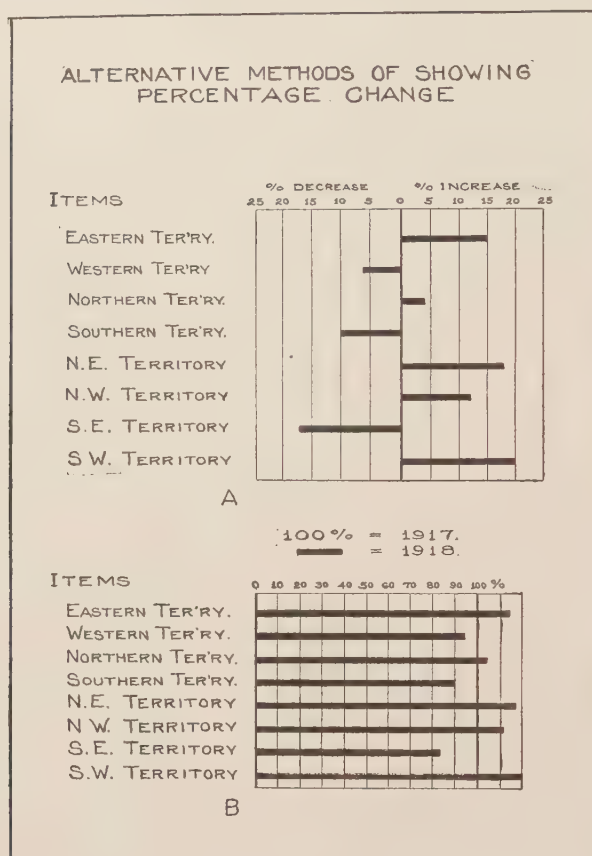


Fig. 79.—Showing loss and gain with horizontal bars.

that has taken place since the previous year. In *A*, simply the increases and decreases are plotted from zero. In *B*, the previous year is considered as 100 per cent, and the percentage that the present year is of the previous year is plotted for each territory. Both methods are clear and accurate but that used in *A* is the most easily understood by the average person.

Systematic Order of Items.—The items represented in any kind of presentation should always be arranged with reference to some systematic order. The most common in size comparisons is the order of size, but it is often preferable to use some other order, such as the alphabetical, geographical, or age. In any case, some systematic order should be used for nothing is lost and both clearness and ease of reading are added.

Comparisons of Differences.—Bar charts are often made to show, not only the differences between the sizes of various aggre-

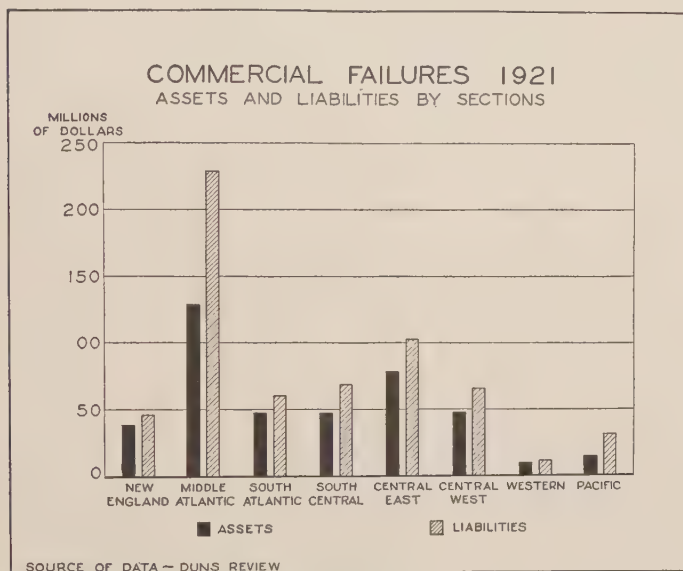


Fig. 80.—Comparing different groups with vertical bars.

gates, but also to compare the differences between certain groups. Figure 80 on commercial failures makes a comparison of assets by sections, a comparison of liabilities by sections, a comparison of assets and liabilities in each section, and a comparison of the differences within each pair of assets and liabilities by sections for the group as a whole. Thus one sees that in New England the liabilities were only about a fifth larger than assets, while in the Middle Atlantic States liabilities were nearly twice as large as assets. These differences can be seen clearly and compared throughout the eight sections.

In Fig. 81 the differences in the types of sales made by the different salesmen may be compared. This could be shown by component bars (see Figs. 89, 90, and 91) which would bring out more clearly the totals sold, but would not show so clearly the actual differences in sales by classes.

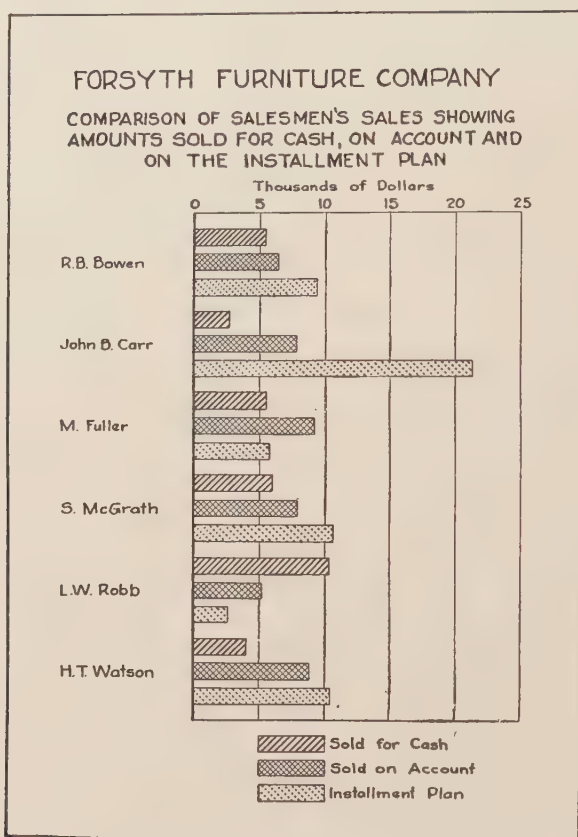


Fig. 81.—Comparing different groups with horizontal bars.

When such comparisons are made, the bars should be arranged in groups as in Figs. 80 and 81. Thus, in Fig. 80 there are two items (assets and liabilities) shown for each of eight geographic sections. In this case the bars should be arranged in pairs, as shown, with enough space between the pairs to separate them distinctly.¹ Then these pairs can be compared and the differ-

¹ A simple method of laying out the irregular spacing of these bars to fit a given space is described on page 11.

ences considered. In order to secure a good effect and a neat appearance, it is better to leave a small space between the bars within each pair or group (Figs. 80 and 81) rather than to place them together. Note that the chart in Fig. 82 is more difficult to read than those in Figs. 80 and 81 because the bars are not grouped or paired for each department. Whatever the arrangement may be in charts like Fig. 80, there should be substantially more space between pairs than within pairs or groups.

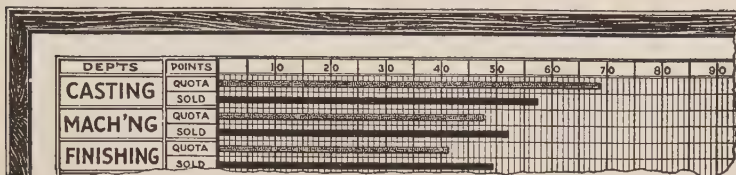


FIG. 82.—Mechanical wall chart for comparing different groups. (Courtesy of the Educational Exhibition Co.)

In such cases, as when two or more different points are being represented for several different departments, it is necessary to so shade or color the bars that they will not be confused. The original chart of Fig. 82 has a green ribbon for quota and a red one for work done. In Fig. 80 the solid bar represents assets throughout the comparison, while the cross-hatched bar repre-

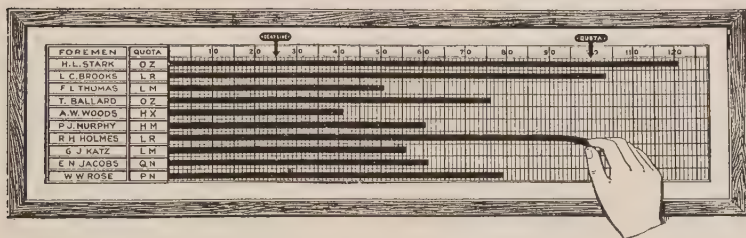


FIG. 83.—Mechanical wall chart. (Courtesy of Educational Exhibition Co.)

sents liabilities throughout, and there is one of each for every section. The explanations of the different kinds of shading or coloring may be made as in Fig. 82, in which all dotted bars are designated "quota" and all solid bars are designated "sold," or they may be explained as in Figs. 80 and 81, in which a key or legend shows what each kind of cross-hatching represents throughout.

Common Unit Necessary.—It is necessary, of course, to use a common unit in making all comparisons of size. It means

nothing to make a chart comparing, for instance, 10 dozen oranges and 50 pounds of prunes. The unit of measurement must be one that is common to both.

Mechanical Bar Chart.—The charts shown in Figs. 82, 83, and 84 are mechanical wall charts. The one shown in Fig. 83 is made in several forms and combinations for showing sales, production, stocks and other data. The bars are movable ribbons, which may be made longer or shorter at will by setting the tab at the ends. The chart shown in Fig. 82 is useful for comparing sales-

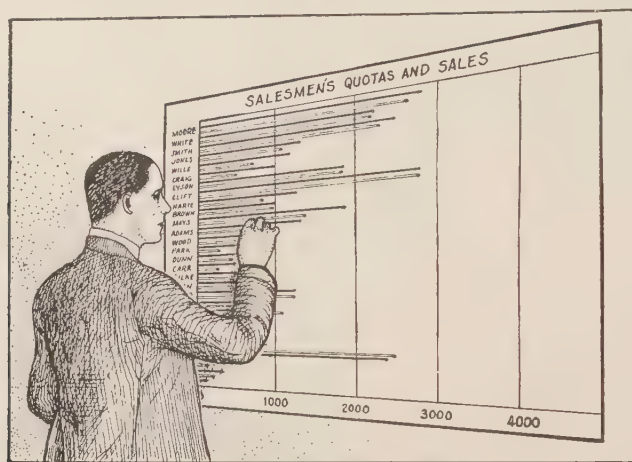


FIG. 84.—A homemade mechanical wall chart.

men's or department's quotas with their actual sales. Different salesmen or departments can be compared in this way, as the chart will show clearly how much the quota is under- or oversold. The large number of background lines is not necessary for reading the chart, but is a convenience for setting the movable bars.

Figure 84 shows a mechanical bar chart which can be made easily and cheaply. This particular chart shows actual sales and quotas of salesmen. Each salesman's name is placed on the left margin, and to the right of this are two small holes made by a pin. Through the top hole runs a red string (black in the illustration) representing the quota, and through the bottom hole runs a blue string (white in the illustration) representing actual sales. The small holes hold the strings and they may be stretched taut across to the tacks at the movable ends of the bars. The board itself is covered with cross-section paper.

Comparisons of the Component Parts of an Aggregate.—Up to this point this chapter has considered comparisons of separate aggregates and comparisons of differences between groups of aggregates. Another important problem in size comparisons is that of comparing the component parts of an aggregate, one with another, or one with the whole. The method illustrated in Fig. 85, commonly called the “pie chart,” probably is the most generally known because of its wide use in popular presentations. This form has been criticized a great deal, on the ground that it is difficult to compare the relative sizes of the sectors, but this is not a serious fault since practically everyone begins early in his

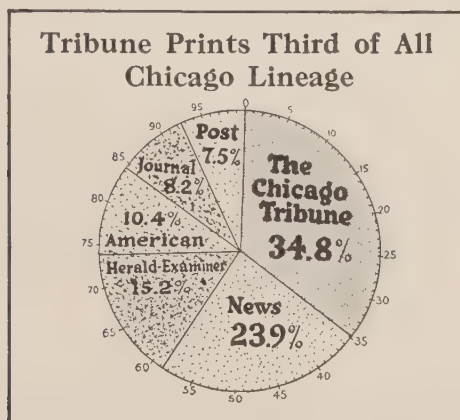


FIG. 85.—A pie chart correctly made. (*Courtesy of Chicago Tribune.*)

career to practice judging the relative sizes of sectors of pies, and is quite familiar with the process of judging such shapes. It is likely, however, that many persons can read a component bar chart (Figs. 90 and 91) more accurately than a pie chart.

The Pie Chart.—The pie chart permits a convenient arrangement of the designations of the different components as well as the percentages and actual values represented. The wording should be placed inside the sectors when possible, and it always should be so placed that it reads horizontally (Fig. 85). The wording never should be placed in such a way that it reads radiating from the center or following the circle as in Fig. 86, for it is much more difficult and confusing to read it than when it is placed horizontally. Figure 86 attempts to show a wide range of extremes and minor subdivisions within the major sectors. Usually these small items are not important enough to be shown

separately, but, if this is desirable, a better method of dividing a sector, or showing components of a component, is to use light or even dotted lines to mark the subdivisions, while the major sectors are limited by heavy lines. Also if a sector must be subdivided, it should be cut by a line radiating from the center, and not by an arc drawn across the sector.

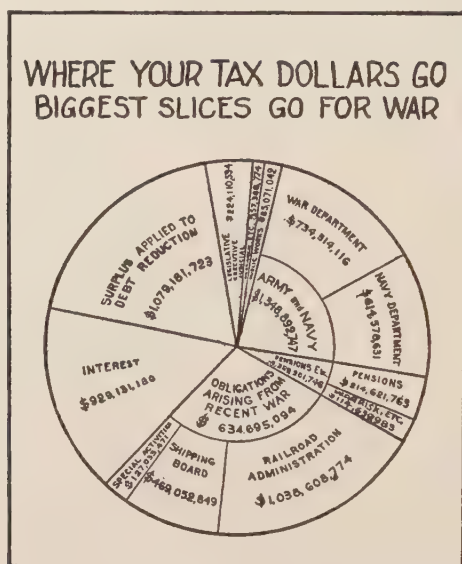


FIG. 86.—A pie chart not correctly made.

If a pie chart is to be most effective, the sectors should usually be made to stand out separately by coloring or shading. Care should be taken to so choose and arrange colors or shading that they will not emphasize the importance of some sectors as compared with others, but will show only the fact that different items are represented. For instance, a solid black sector will stand out more prominently than a cross-hatched sector of the same size, or a bright red will stand out more than a dull gray. The size alone of the sector should signify its importance.

The sectors of a pie chart should be arranged according to some systematic order. Usually they are arranged in order of size, and the most generally followed order is to arrange them as in Fig. 85, in which the order is from large to small in a clockwise direction beginning at the top. Of course, a sector representing a combination of several small items, and designated "all others,"

naturally would be given last place even if larger than some of the other sectors.

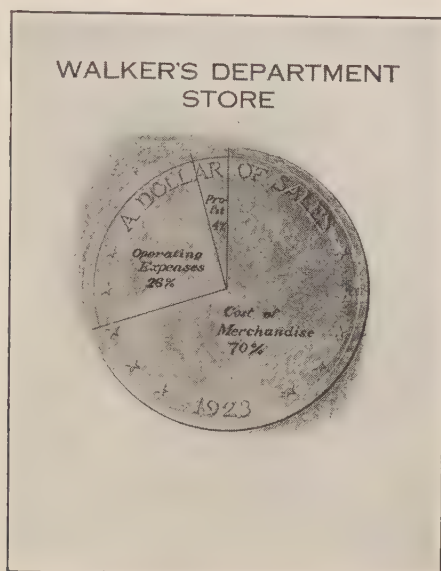


FIG. 87.—A pictorial pie chart.

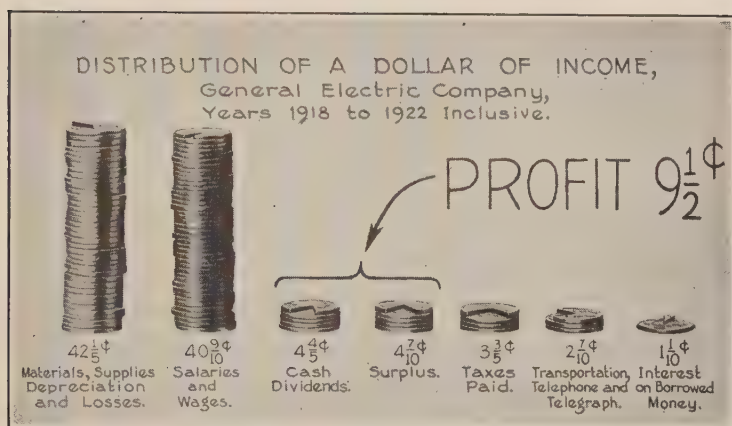


FIG. 88.—Another pictorial method of showing data similar to that of Fig. 87.
(Courtesy of General Electric Co.)

In making popular presentations, pictures are sometimes placed on the sectors. When using this method, one should not make the illustrations too prominent because the eye may compare

the sizes of the illustrations rather than the sizes of the sectors. This type of practice should be followed only in the most simple forms of charting, such as those used in certain popular magazines and newspapers or in children's books and magazines.

In making an appeal to certain types of business men, a very effective application of the pie chart principle is shown in Fig. 87. Figure 88 shows another presentation of similar figures which is easily understood by the average reader. It is simply a bar chart made up in popular form to represent the component parts of a dollar. The ordinary bar chart is often used to show component parts in this way.

The Component Bar Chart.—An excellent form for showing the component parts of a whole is the component bar (Figs. 89,

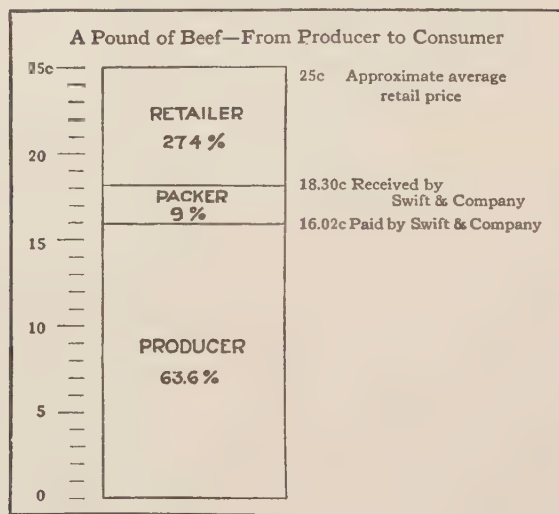


FIG. 89.—A vertical component bar. (Courtesy of Commercial Research Department, Swift and Co.)

90, and 91). The component bar may extend either vertically or horizontally, but the direction should be chosen that will make the chart most easy to read. The designations of the different components may be placed within the bar (Figs. 89 and 90) or outside the bar as in Fig. 91. It is well to show scales on these charts as a further aid in reading and checking (see Figs. 89 and 91). If designations are placed outside a vertical bar, it is usually best to place them at the right of the middle of each component as in Fig. 91. This leaves the left side free for a scale, while designa-

tions of uneven length will be close to their components and still begin in a straight vertical column.

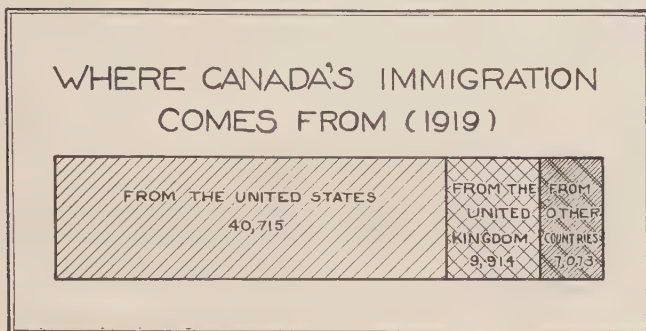


FIG. 90.—A horizontal component bar.

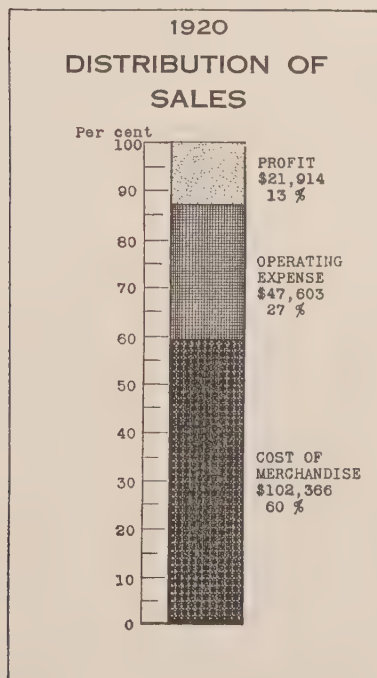


FIG. 91.—A vertical component bar with scale.

The components should be arranged in a systematic order such as size. If the order of size is followed, a vertical bar will appear

less top-heavy if the larger sections are at the bottom. In Fig. 89 the order of size was not used since the packer naturally comes between the producer and the retailer.

The discussion of coloring and shading pie chart sectors applies equally well to bar components. It is well to differentiate the components in this way, but care must be taken not to emphasize some of them by using strong cross-hatching or brilliant colors.

Complicated Forms Should Be Avoided.—Figure 92 is a combination of a component bar and simple bar chart. The number

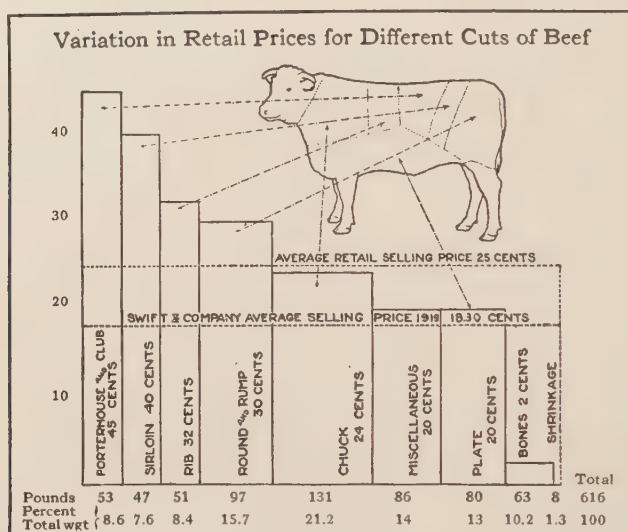


FIG. 92.—A complicated form. See Fig. 93. (Courtesy of Commercial Research Department, Swift and Co.)

of pounds of the different cuts of beef are shown in the horizontal component bar. The vertical length of each component represents the price of the cut. The area of each rectangle would represent the total amount received for the cut. This chart, however, is not easy for the average reader to understand, and the same figures for the following year were represented as in Fig. 93, which tells the story very simply and clearly. An objection to the plan used in Fig. 93 is that the bars representing pounds extend to the left of the base line, and this practice should be reserved for showing decreases, losses, and other negative

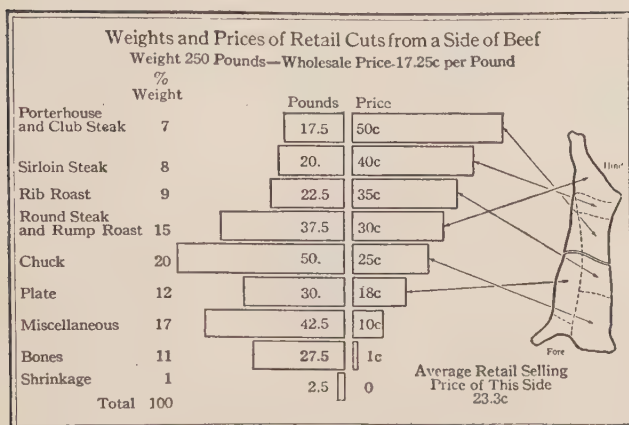


FIG. 93.—A better form for showing the data of Fig. 92. Note, however, that the practice of extending bars to the left of the zero base should be reserved for showing minus quantities. See Fig. 79A. (Courtesy of Commercial Research Department, Swift and Co.)

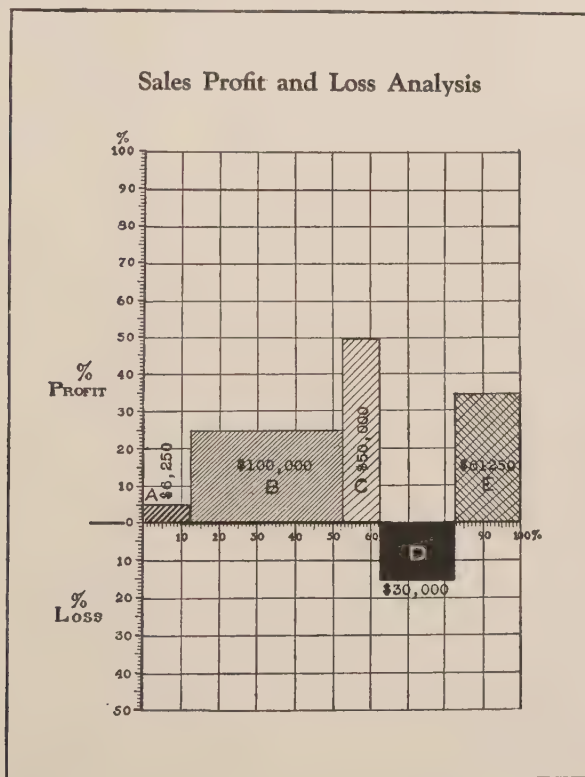


FIG. 94.—Complicated forms like this should be avoided.

quantities. Horizontal bars representing positive figures should extend to the right of the zero base.

In the sales profit and loss analysis chart shown in Fig. 94, the aim is to give a picture of the earnings of the different types of merchandise. *A*, *B*, *C*, *D*, and *E* are commodity symbols of the various classes of commodities represented. The vertical scale shows the percentage of profit or loss on each commodity. The horizontal scale shows the volume of sales of each commodity expressed as percentages of the total. The area of each rectangle represents the amount of profit or loss on each commodity. An important objection, however, to this chart is that it is hard to

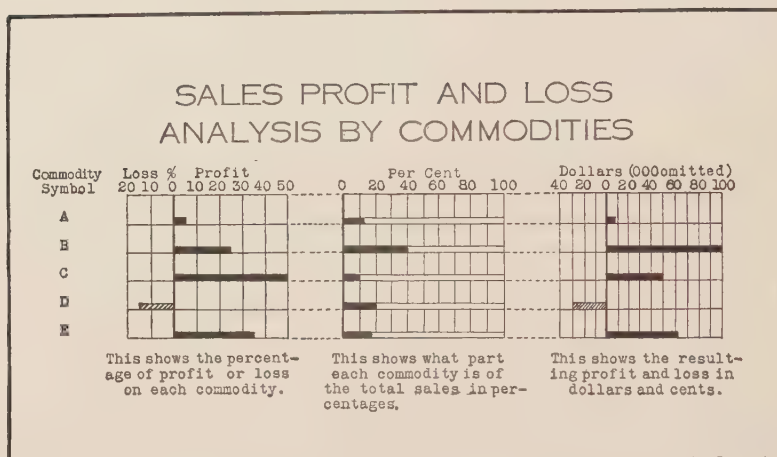


FIG. 95.—A more simple method of showing the data of Fig. 94.

compare such irregular areas. Another form of making the same comparison is shown in Fig. 95. This form is more simple in construction than Fig. 94 but it has three times as many bars. As a general rule, however, it is well to keep away from the use of forms that involve the comparison of areas if it is possible to use forms that vary only in one dimension. Note that the percentage of the total is expressed in the central group in Fig. 95 by indicating 100 per cent in outline and showing the percentage for each commodity in solid black. The loss figures are shown in red on the originals of both Figs. 94 and 95.

Pictorial Forms for Showing Component Parts.—In Figs. 96, 97, and 98 the aim is to add interest and clearness by picturing

Why It Costs So Much To Mine Anthracite

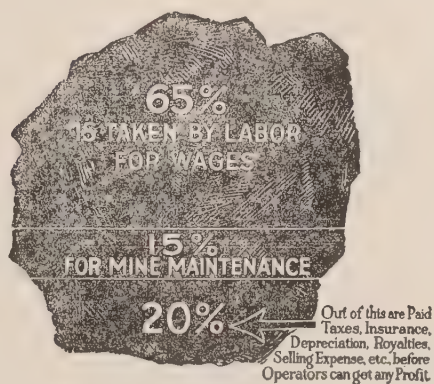


FIG. 96.—A pictorial component bar. (Courtesy of Anthracite Operators' Conference.)

HOW THE AVERAGE AMERICAN FAMILY SPENDS ITS DOLLAR

Data from the Eighteenth Annual Report of the Bureau of Labor



FIG. 97.—A pictorial component bar.

the thing represented as well as to show the various component parts. The irregular shape of the lump of coal in Fig. 96 does

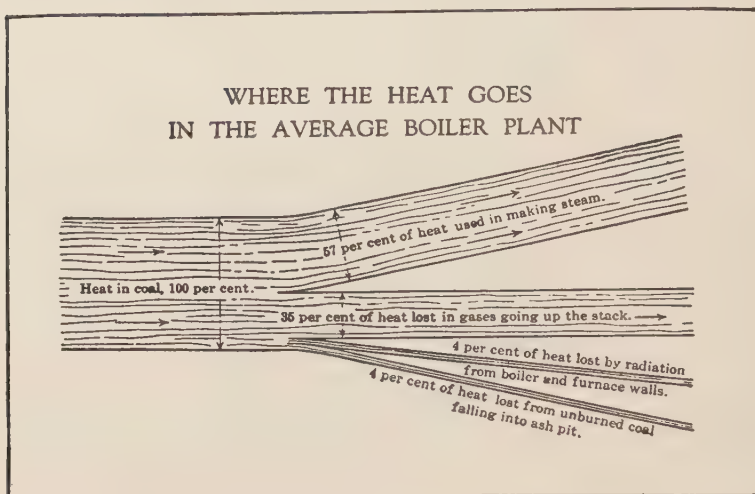


Fig. 98.—A pictorial component bar. (Courtesy of National Engineer.)

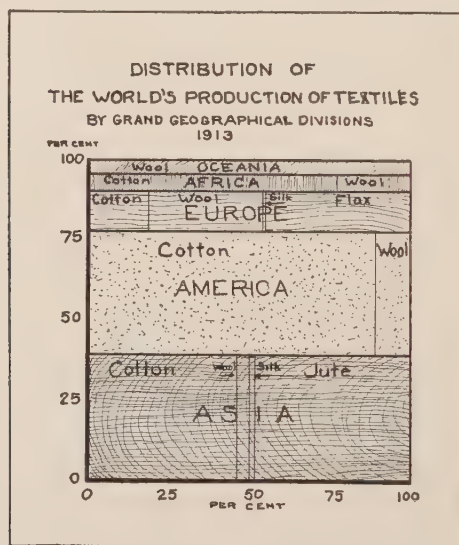


Fig. 99.—A poor method of showing component parts of component parts. Compare with Fig. 100.

not permit easy or accurate measurement by the reader, but it is quite effective in showing the divisions roughly and this is

the purpose of the chart. For popular reading Fig. 97 is a good presentation to show roughly the distribution of living expenses. Figure 98 is an effective presentation of the relation of heat used in making steam to the rest of the heat in bituminous coal.

Component Parts of Component Parts.—A method of showing component parts of component parts is shown in Fig. 99. If a component must be subdivided, this method is effective only for presentation to a trained reader. Since this problem is not a common one, the form shown in Fig. 100 is better for general use.

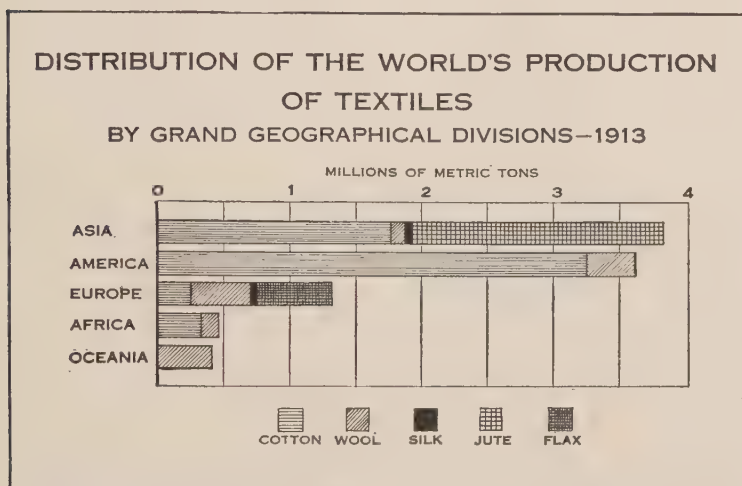


Fig. 100.—Usually a better method of showing the data of Fig. 99.

Here the major components are separate bars which show the subdivisions representing minor components within each bar.

Comparisons of Aggregates Made Up of Components.—Up to this point the discussion of component parts has considered only the comparison of those parts which are within single aggregates. Often it is desirable to compare aggregates made up of component parts either with reference to the actual sizes of the aggregates or with reference to the percentage distribution of the parts within the aggregates. If the problem is one of comparing the percentage relations of the parts to the whole, either the pie chart or the component bar chart can be used, as the different forms representing 100 per cent would be the same size (Figs. 101 and 102). When using the bar form in careful work,

it is best to show a scale and a background of guide lines as in Fig. 102. If, however, the problem is one of comparing actual sizes of aggregates and their components, the bar method should



FIG. 101.—Use of pie charts in comparing the percentage distributions of the component parts of different aggregates. (*Data fictitious.*)

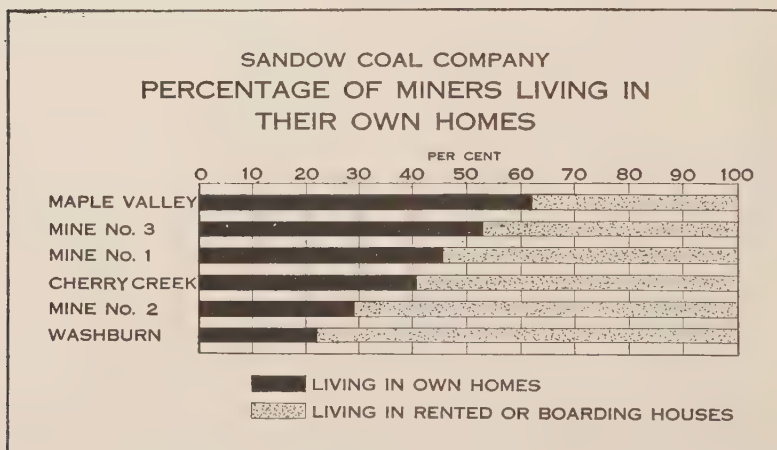


FIG. 102.—Use of the bar chart in comparing the percentage distributions of the component parts of different aggregates. (*Data fictitious.*)

be used in preference to the pie chart. Circular areas (Fig. 103A) are hard to compare, while this difficulty does not arise in using component bars (Fig. 103B).

In both percentage and actual value comparisons, the components should be differentiated by shading or coloring—each shade

or color representing the same thing in all the bars or circles. The designations may be placed on the components, as in Fig. 103, when only a small number of aggregates are compared. When several items are compared, it is easier to read the chart if a key or legend descriptive of the cross-hatching or coloring is placed on the chart as in Fig. 102.

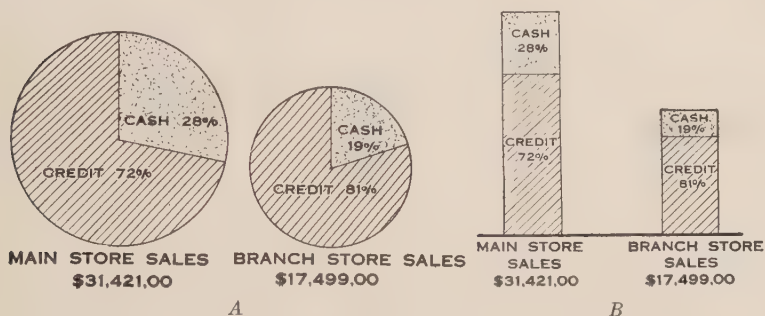


FIG. 103.—Comparison of aggregates which show component parts. *A*, Incorrect, as sizes of circles are difficult to compare. *B*, Correct, as it is easy to compare bars of different lengths.

Figure 104 illustrates a method of comparing distributions that is very easy to understand. The relation of one pie chart to the other is clearly indicated by the descriptive matter and arrows between the charts.

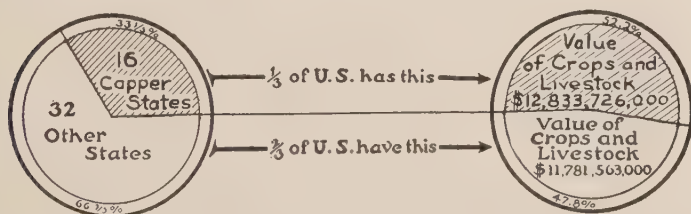


FIG. 104.—Comparison of percentage distributions for popular presentation. (Courtesy of Capper Farm Press, Copyright, 1920.)

A Common Error in Using Component Bars.—The purpose of these charts and the nature of the data represented must be clearly understood and should not be confused, as is the case in Fig. 105 where the black portion was placed over the cross-hatched portion simply because the asset figures happened to be smaller throughout the comparison and not because they were component parts. In this figure an amount represented by

cross-hatching is measured from the zero line, and it is not easy to imagine this shading extending from the zero line with the solid black shading superimposed upon it. Assets and liabilities are separate aggregates and should have been shown by separate bars as in Fig. 80.

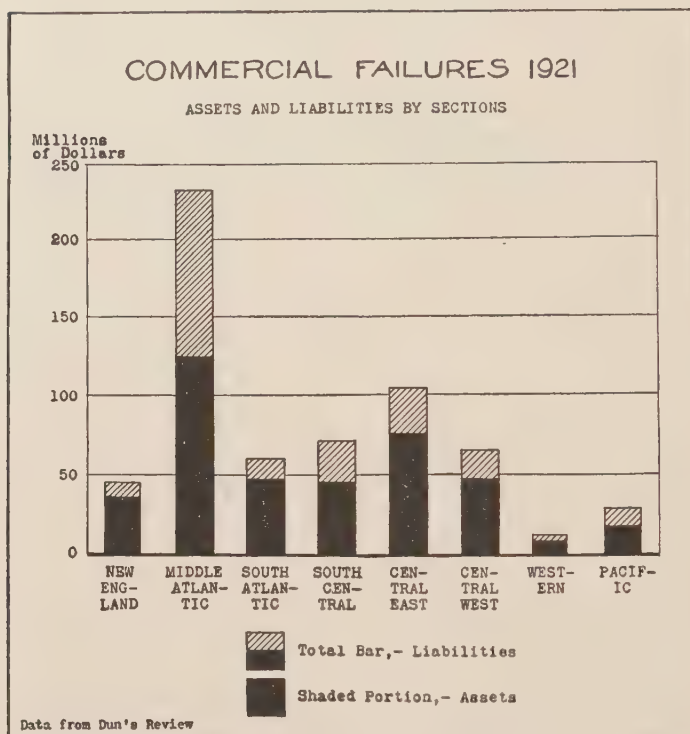


FIG. 105.—A common error. Incorrect as the chart does not show component parts.

Checking List.—The following list includes the most important points to be observed in making the charts described in this chapter, and can be used when checking a chart to see if it is complete:

Did you check for accuracy?

Is the title simple and clear? Did you place it at the top of the chart?

If you have used shadings or colors to make any necessary distinctions, have you included a legend or key to the meanings of the shadings or colors?

Have you placed a convenient scale and a background of lines upon the chart to aid in measuring sizes?

Are the faces of the bars free from these lines?

Is the zero line heavy?

Are the other scale lines light?

Have you designated the scale units?

Does all lettering read horizontally? (When impossible to letter horizontally, lettering should read from bottom to top—never from top to bottom.)

Are your letters formed according to a commonly accepted style of plain lettering?

Have you given the reference or source?

Have you ruled in a border line? (Optional but advisable.)

If desirable, have you included your name (lower right) and date (lower left)?

Are pencil lines erased?

Will your chart be clear to the reader?

CHAPTER V

HISTORICAL TIME SERIES

It is the purpose of Chapters V and VI to describe the graphic methods of presenting data in which time is a factor. Probably the most important field in business statistics is that in which phenomena are compared at different *dates*. Thus, we may wish to compare our sales for the last month with the sales of every other month for the past 5 years in order to determine whether or not we are gaining or losing trade. By presenting these figures graphically, we are able to see at a glance a picture of the whole period.

This study of presenting time series is divided into five distinct parts as follows:

1. Size changes of a single variable over a period of time.
2. Comparisons of size differences between different time variables.
3. Components of a time variable.
4. Rates of change in a single variable.
5. Comparisons of rates of change between two or more variables.

Number 1 above would be illustrated by a chart of a single series showing, say, amounts of sales by months. Then the part of the curve for the last month could be compared with the parts of the curve showing amounts of sales in the same business for other months. Number 2 would be illustrated by a chart of more than one series comparing the volume of sales of the main store with the volumes of sales of, say, its branches for the same period. In number 3 we might show over a period of time our sales of, say, shoes and how they were divided into men's, women's, and children's, or we might show how the total was divided into parts sold by each salesman. Number 4 would show, for instance, whether our sales are changing at an increasing or decreasing *rate* as compared with the past. In number 5 we could compare the main store with its branches, not to show how much larger the one is than the others, but to show by the slopes of the curves which store is increasing its sales at the most

rapid rate. These five parts are distinct forms and should be recognized as such. One cannot be used safely unless the others are understood, and one cannot take the place of another. They will be considered in the order given above.

SIZE CHANGES OF A SINGLE VARIABLE OVER A PERIOD OF TIME

In this kind of comparison the problem is to show the sizes of the phenomena at different dates. Thus, we might compare

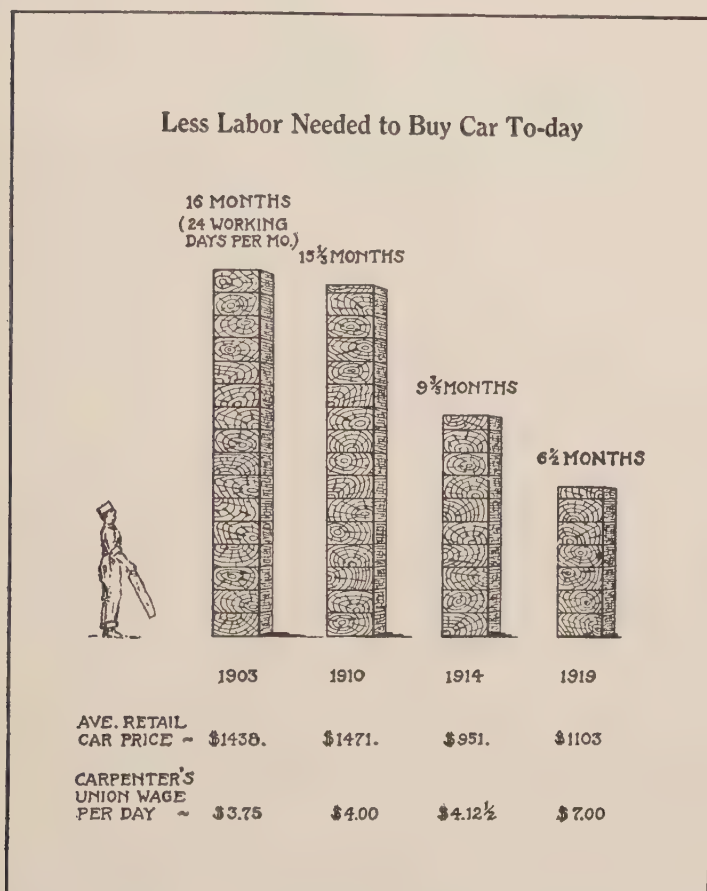


FIG. 106.—Pictorial chart of time series. (Note that unequal time intervals are sometimes used in pictorial charts.) (Courtesy of National Automobile Chamber of Commerce.)

the amounts of labor needed to buy a car at different dates as shown in Fig. 106. The study is confined to a single series.

Simple Pictorial Time Charts.—When the problem is one of popular presentation, the pictorial chart is very effective, but in all cases a geometric form should be chosen which may be made



FIG. 107.—Pictorial comparison of two time periods. (Wrong because volumes are misleading. Also chronological order is reversed.)

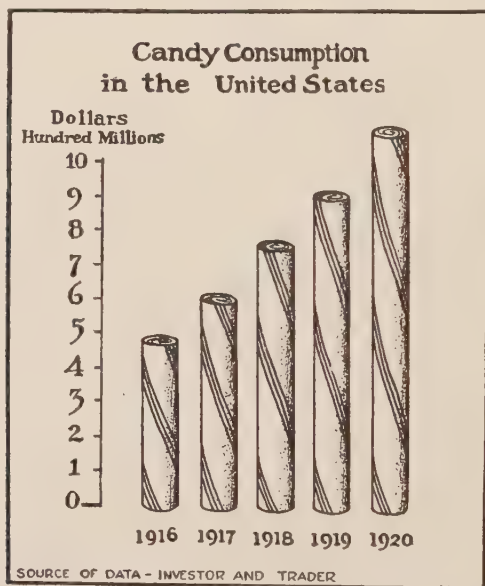


FIG. 108.—Pictorial presentation of time series. (Correctly drawn.) (*Adapted from Investor and Trader.*)

to vary in a single direction without distortion. Thus, in Fig. 106 the comparison is clearly made since the piles of lumber vary only in their heights. In Fig. 107 the amounts of gold mined in

the United States in 1920 and 1915 are compared. The amounts represented are proportional to the heights of the piles. As the eye unconsciously compares the volumes, the reader gains a wrong impression of the change. The large pile represents twice as many ounces as the small one yet it contains much more than twice as much gold dust. Volumes, varying in three dimensions, should never be used in graphic presentation as they are quite likely to be misinterpreted. Figure 108 represents the amounts

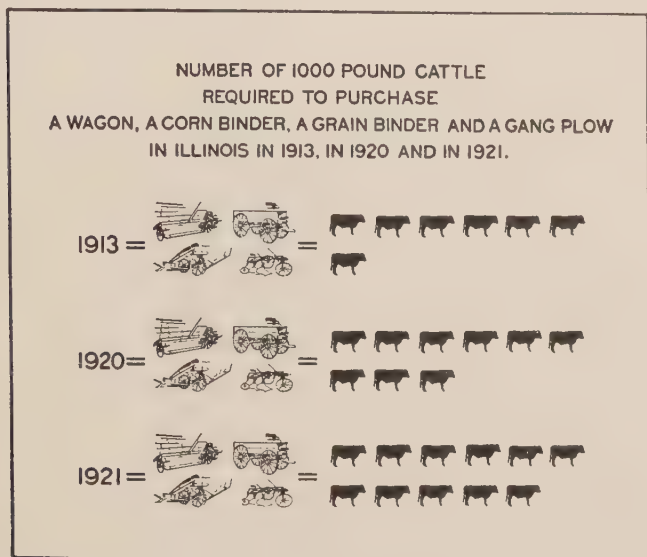


FIG. 109.—Pictorial chart of time series for popular presentation. Pictorial charts are sometimes exceptions to the vertical scale rule stated in connection with Figs. 110 and 111. (*Courtesy of United States Department of Agriculture.*)

of money spent for candy during a 5-year period, and is an effective chart for popular presentation.

Ordinarily the time scale should be laid out along the horizontal axis of a time chart, but occasionally it may be laid out vertically in popular presentations as illustrated in Fig. 109. This rule, however, should not be violated in any but pictorial popular charts and then only when distinctly necessary.

When planning a chart, it should be remembered that for the trained reader the pictorial effect adds nothing. It is, however, an excellent method to use in popular publications and in children's books and magazines.

Bar Charts for Time Series.—Bar charts, similar to those described in the preceding chapter, are often used to show time

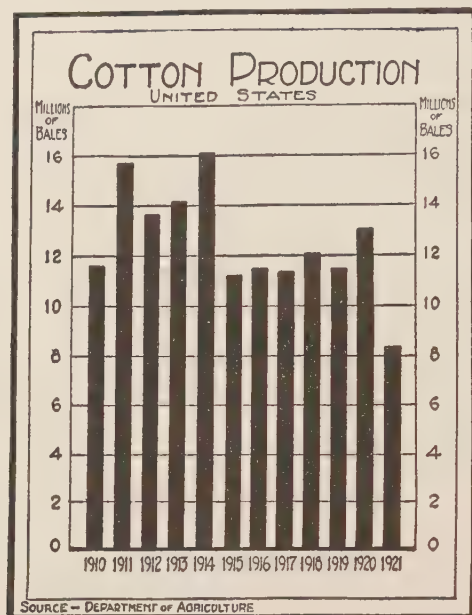


FIG. 110.—Vertical bars—the correct form of bar chart for showing time series.
(Courtesy of Federal Reserve Bank of Philadelphia.)

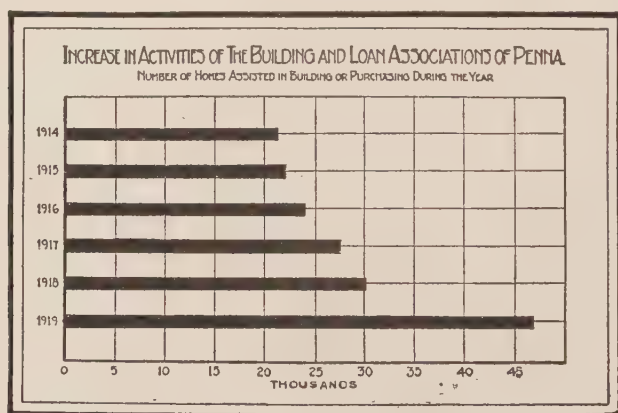


FIG. 111.—Horizontal bar chart. Incorrect for showing time series. (Abbreviations should not be used in titles)

series (Figs. 110 and 111). In showing time series, however, it is much better practice to use the vertical bar chart (Fig. 110),

than the horizontal bar chart (Fig. 111).¹ The reason for this is that the chart of vertical bars is read in exactly the same way as a time curve, which is by far the most common way of presenting these data. When the vertical bar chart is used for showing a series of positive figures over a period, the time scale should read

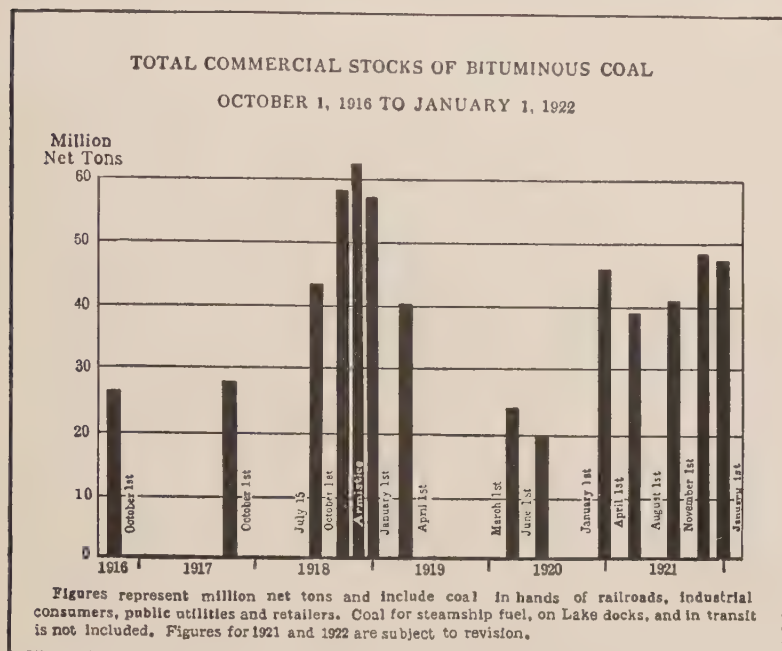


FIG. 112.—Correct use of bar chart for showing incomplete series. (Courtesy of United States Geological Survey.)

from left to right as shown in Fig. 110. This scale should be laid out in such a way that equal spaces represent equal time intervals over the period. That is, for instance, each inch might equal 1 year. This rule should be followed even though the data are not complete as in Fig. 112. Had the bars been evenly spaced in this figure, a wrong impression would have been gained by the reader. It should be noted that this rule is sometimes violated in pictorial charts which are used in popular presentations (see Figs. 106 and 109).

¹ This does not apply, however, in making the simple comparisons of size described in Chap. IV, see p. 50.

Curves of Size Changes.—The most common method of showing time series graphically is by means of a curve.¹ In Fig. 113 the arithmetic time curve tells the same story as the bar chart and the similarity is illustrated.² A curve represents the ends of the bars, but by being continuous it indicates the direction of the change more clearly than a bar chart.

The general arrangement of a time curve should be such that it reads from left to right. Thus, as in part *C* of Fig. 113, we

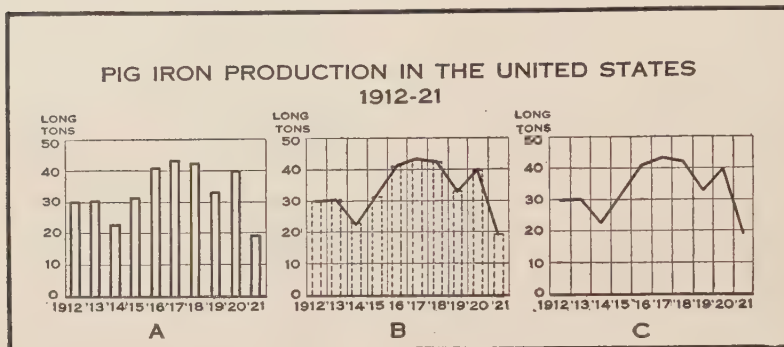


FIG. 113.—Comparison of bar and curve methods of showing time series.

follow the curve over the period in the direction in which we naturally read. As far as possible the lettering should be so placed that it can be read horizontally from left to right. When it cannot be so placed, it should read from bottom to top.

The Arithmetic Vertical Scale.—In the vertical scale for an arithmetic time curve, equal spaces represent equal values. Thus, in the original of Fig. 114, a quarter of an inch represented \$5 anywhere in the scale. Note, however, that expressions such as " $\frac{1}{4}$ inch = \$5" are not placed on statistical charts. All necessary measurements can be read much more easily from the



¹ A very essential element in graphic presentation of statistics is the systematic use of *coordinates*, that is—numbers which determine the position of a point in a plane. *Coordinate axes* are two indefinite straight lines at right angles to each other. Their point of intersection, *O*, is called the *origin*. The line *OX* is called the axis of *X* or the axis of *abscissas*. The line *OY* is called the axis of *Y* or the axis of *ordinates*.

² This type of curve is referred to as an arithmetic time curve to distinguish it from other types of curves (logarithmic time curve, frequency curve, and correlation curve) which will be discussed later.

scale on the graph, and in photographic enlargements or reductions the scales are taken care of automatically.

The correct position of the vertical scale is on the left of the first ordinate (Fig. 114). If it will add anything to the ease of reading the chart, a scale may be placed on the right of the last ordinate also, but in no case should the scale be omitted on

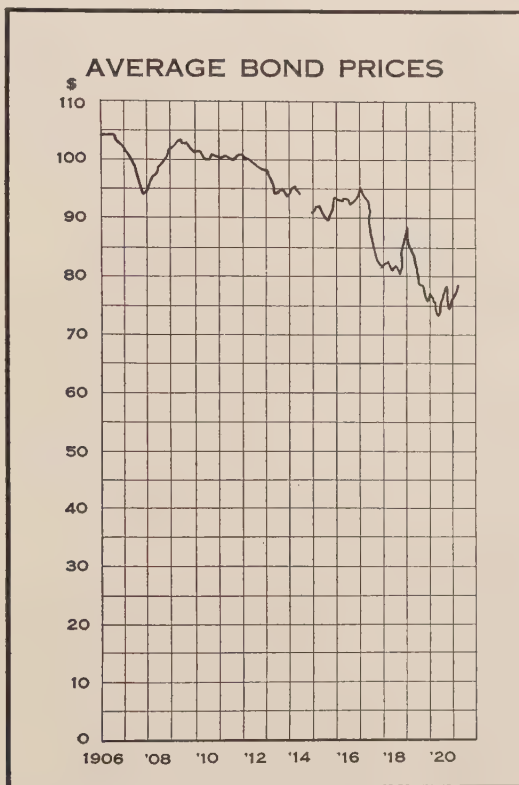


FIG. 114.—Arithmetic time curve. (Correct scale.)

the left. The figures of the scale should be exactly opposite the lines which they represent (Fig. 114). The units in which the scale is laid out should be explained by a caption at the top of the scale.¹

Importance of Including Zero on the Vertical Scale.—When the problem is one of comparing sizes over a period of time, the zero line and all of the scale must be included on the graph. If

¹ See the discussion of captions in the preceding chapter, p. 54.

the lower part of the scale is not shown as in Fig. 115, the reader is given either no definite impression of the size changes, or a very inaccurate one. The effect is likely to be that the prices represented have decreased much more than they have in fact, for only by referring to the scale will it be seen that the prices have decreased only about one-fourth. This correct impression is given only when the whole scale is included as in Fig. 114, where the decrease appears in its true relation to the rest of the period. If it is desired to use only the space covered by Fig. 115, the vertical scale of the chart in Fig. 114 may be contracted to fit, but the base should not be cut off in size comparisons. (This

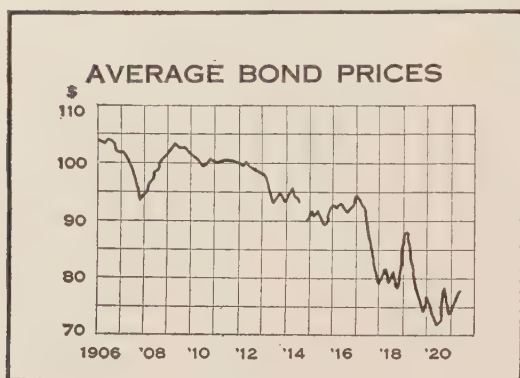


FIG. 115.—Incorrect because lower part of vertical scale is cut off. Compare with Fig. 114.

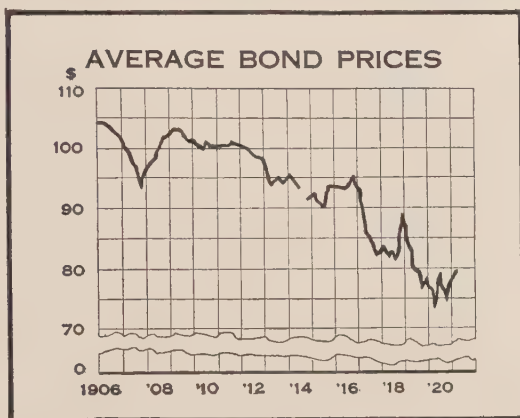
principle was discussed also in Chap. IV in connection with Figs. 67 and 68.)

Occasionally it is desired to show time changes without considering the size or rate of change. For instance, suppose that in Fig. 114 the purpose is to show the fact that prices have started upward without considering either the amount or the rate of the change that has taken place. In such cases, where the entire scale does not appear, attention should be called to the fact by showing the zero line below a horizontal break in the graph as in Fig. 116A, or simply by showing a break at the bottom of the graph as in 116B. Either of these warnings should prevent the reader from attempting to compare sizes for the different dates.

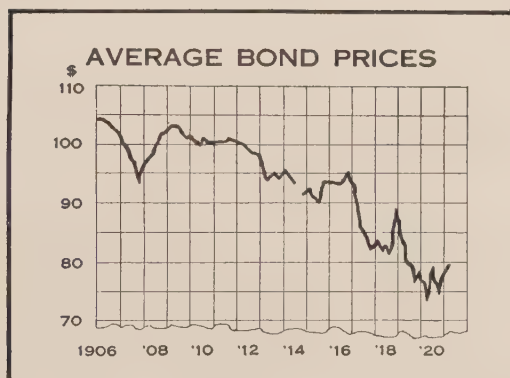
Figure 117 is plotted in percentage relatives,¹ with the 100 per cent line as the base above or below which the curve is plotted.

¹ Percentage relatives of values at given times are the percentages that these values are of a value for a certain period taken as a base of 100.

The zero line of a percentage relative scale may be omitted if it is relatively a long distance from the 100 per cent line. If, however, the zero line is included, it is much easier to see the relative values of the points. For instance, if the whole scale is given, and the curve drops to 75 on the scale as in Fig. 117, it is easier



A



B

FIG. 116.—If the entire scale is not shown, this fact should be emphasized by a break at the bottom of the chart, as in A or B above.

to appreciate that the curve has dropped one-fourth of the way to zero. In Fig. 117 it is stated that the latest figure is for March. This is a good plan on charts with time scales which do not show individual months clearly.

The Horizontal Scale.—The most commonly accepted position for the horizontal scale is along the bottom of the graph as shown in Figs. 114, 118, and 123. If it will make the graph easier to

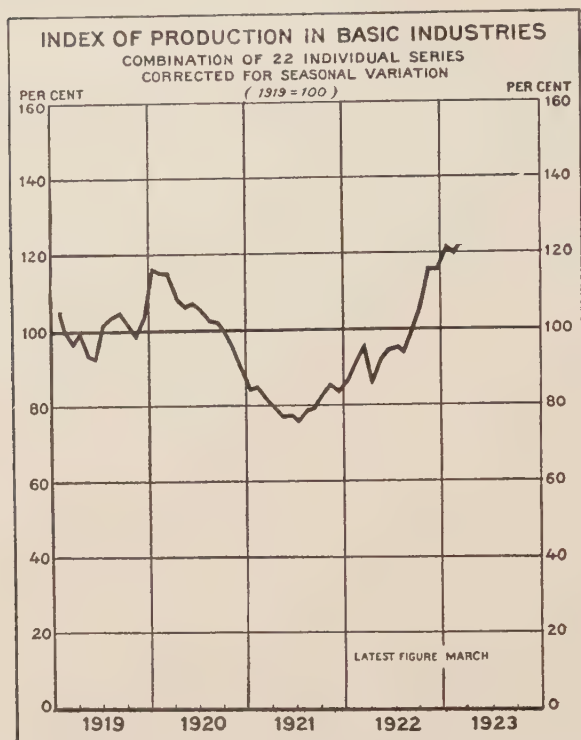


FIG. 117.—Curve of percentage relatives. (Courtesy of Federal Reserve Bulletin.

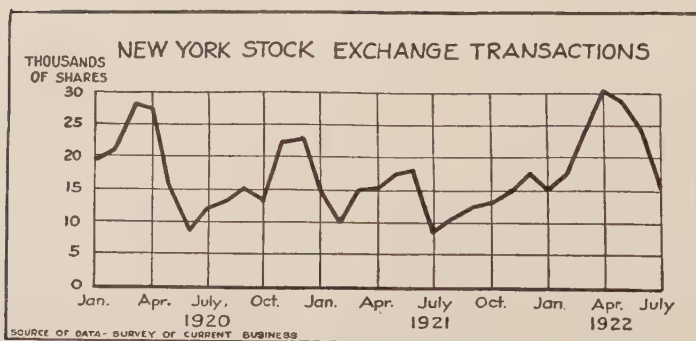


FIG. 118.—Designating months by quarters. (See footnote, page 91.)

read, a scale may be placed along the top in addition to the one along the bottom. However, the scale along the bottom should never be omitted. By standardizing this position, charts are easier to read, since the designations are in the most convenient position for making comparisons from the zero base line. It is not necessary to explain in what units the time scales are expressed, except when hours, days, or some unit is used which is not clearly recognized. Years or months are always clear without an explanation or designation. In making charts for general use, it is better to designate months by their common abbreviations rather than by their first letters, which are confusing because the names of more than one month begin with the same letter. If there is not room to letter all the abbreviations horizontally, it usually will mean a clearer chart if only the quarterly months are designated as in Fig. 118, in which the data are plotted monthly and the ruling and designations are on a quarterly basis. In designating by quarters, some statisticians use the first month of each quarter (January, April, July, October), while others prefer to use the last months (March, June, September, December).¹

No arrows, dimension lines, or other guides or boxes are necessary to show the relation of months to years. In fact, no lines should be included which are not necessary in reading the chart. In showing data by weeks, the monthly divisions are confusing if ruled across the chart, but they may be shown if desired by adding short division marks vertically above or below the zero line.

Whether points should be plotted on the lines as in Fig. 119A, or between the lines as in 119B, is a matter to be decided by personal preference. In either case, the designation of the month

¹ Some statisticians hold that in a chart like Fig. 117 the line at the end of a space representing a year should carry the figures for December (as shown) instead of January. Otherwise it is claimed that the space as a whole does not represent a year. Either plan will serve, however, when it is remembered that in a non-cumulative chart, like Fig. 119A, the space between lines does not represent a period of time, but the line itself represents the whole month. If the curve is cumulative, then not only the month but the day of the month should be given (see Fig. 126). Whatever method is used, the chart should be clear to the reader. The practice here should be decided by an international agreement arrived at by the Royal Statistical Society, the American Statistical Association, and other similar organizations. At the present time, however, practice seems to tend toward the establishing of a convention that the line at the end of a space representing a year shall carry the figures for December.

(or other period) should be placed exactly below the point plotted in order that there may be no confusion. Thus, in Fig. 119*A* each name of a month is directly under the vertical *line* on which the point is plotted. In 119*B* each name is directly under the *space* in which the point is plotted.

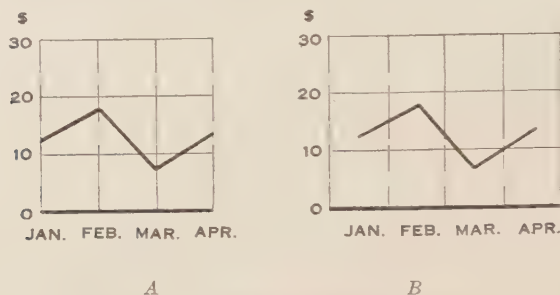


FIG. 119.—Position of points plotted.

In the horizontal scale, equal spaces should represent equal units of time. In this respect Fig. 120 is wrong and misleading because the spaces representing years vary in length. In this

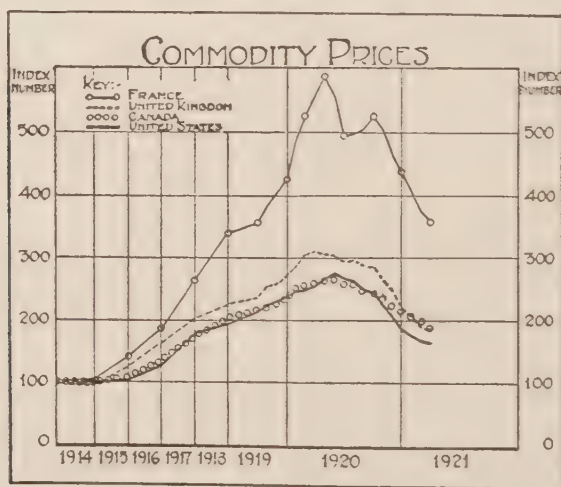


FIG. 120.—Unequal spacing for years. (Incorrect.)

figure the points were plotted annually from 1914 to 1918 inclusive, 1919 was plotted semiannually, and 1920 was plotted quarterly. It would not be wrong to plot in this way if the years were represented by equal spaces throughout the period, but it is

wrong to give 1920 twice as much space as 1919 and four times as much as the other years.

In a chart like Fig. 121 a distinct break should be shown between 1916 and 1919 where 2 years are omitted. Otherwise, the effect is likely to be misleading even though there is a notation at the bottom of the chart. This chart is misleading in another respect as the scale does not begin with zero.

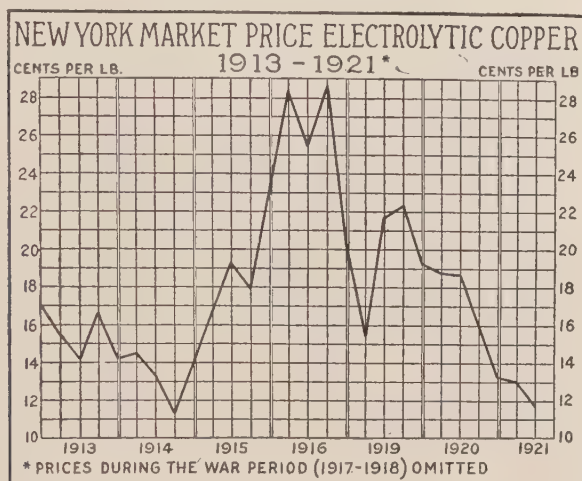


FIG. 121.—Omission in horizontal scale not represented graphically. (Incorrect.)

Emphasis of Lines.—The zero line should be made heavy in order that it may be sharply distinguished from the other coordinate lines. Usually it is desirable to emphasize the 100 per cent line (Fig. 117), or any other line that is used as a base for the comparison. In the horizontal scale of a time graph it is usually best not to emphasize the first ordinate since, as a rule, it is no more important than the other ordinates. Any line may be emphasized, however, for the definite purpose of adding clearness to the graph, but no lines on the graph proper should be emphasized merely for decorative purposes.

The curve on a graph should be heavier than the coordinate ruling, as it should stand out clearly on the background. The curve and the zero line should be prominent since they define the limits of the sizes represented, that is, the zero line defines the bottoms of the sizes represented and the curve defines the tops. Since our interest is centered chiefly in the curve, it

may be made heavier than the zero line. The background of coordinate lines should be very light as compared with the curves and zero lines (Figs. 122B and 123).

Scale Units and Number of Coordinate Lines.—The units of the scales should be given in round numbers of intervals con-

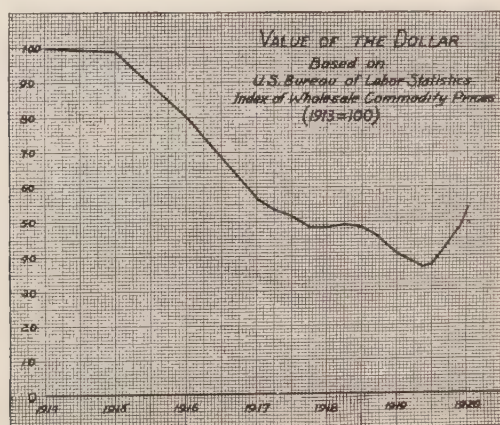


FIG. 122A.—Too many coordinate lines in background.

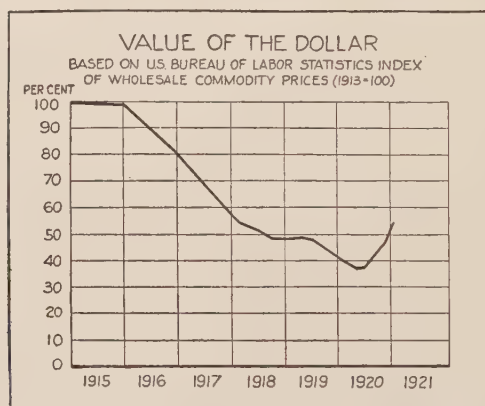


FIG. 122B.—An improved form for showing Fig. 122A.

venient for reading. No more scale units or coordinate lines should be shown on a graph than are necessary to aid the eye in reading the curve. Just how many to show is a matter for the chart-maker's judgment and will be learned with experience. However, it is easy to see the difference in effect by comparing A

and *B* in Fig. 122. Figure 122*B* has all of the lines that are necessary for easy reading, while the large number of lines in 122*A* adds nothing of value for making visual comparisons.

Showing Plus and Minus Quantities.—Positive numbers are shown above the zero line, and if negative numbers occur in the same series they may be represented along with the positive by plotting below the zero line as shown in Fig. 123. In the original of this chart (123), profits are in black and losses in red. Blocking in the areas between the curve and the zero base, as in Fig. 124, makes the chart still more effective for some purposes.

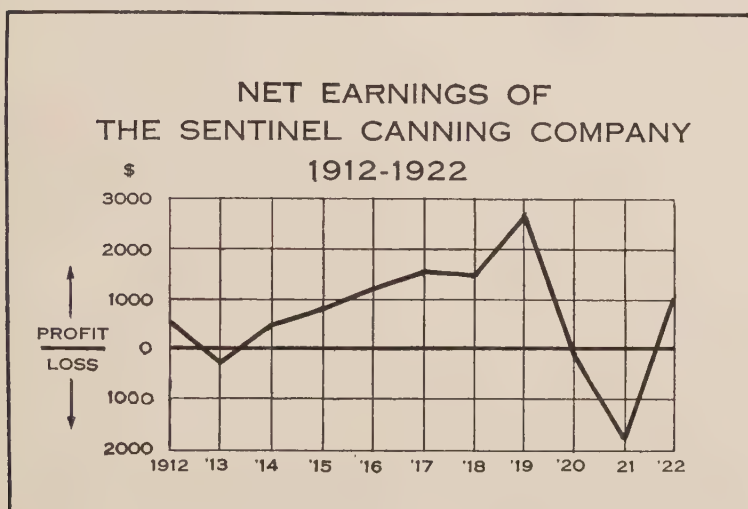


FIG. 123.—Showing positive and negative variations in a single variable.

While the areas are very irregular, it is easy to see that according to this chart the farmer's losses on hogs are much larger than his gains, taking the period as a whole.

Smoothed and Broken Curves.—In charting non-cumulative data, the original points plotted should be connected by straight lines. All that the lines between the points signify is the direction from one point to another.

For instance, in Fig. 125 each ordinate represents a month. The first one represents the entire month of January, the second represents February, and so on. There is no time represented between the lines and there is no value on the curve between the plotted points. The point on the February ordinate represents the total for that month, but halfway on the curve between

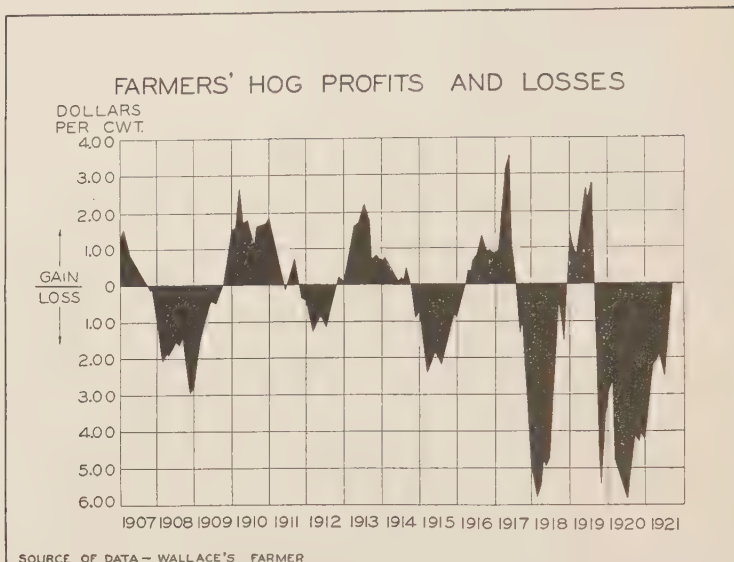


FIG. 124.—Use of shading to emphasize variations. Compare with Fig. 123.

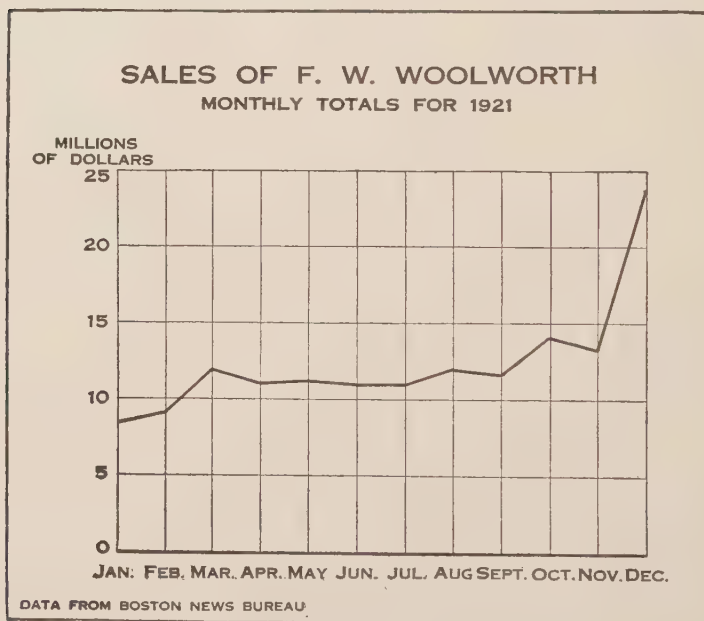


FIG. 125.—Broken curve, (Non-cumulative.)

January and February would not represent the sales for the middle of the month. All that the curve does is to point the way from one total to the next. However, considering the period as a whole, the area under the curve represents the sales over the year.

Figure 126 shows the data of Fig. 125 plotted on a cumulative basis. The space from one month to the next now represents a period of time, and every point on the curve now has a value.

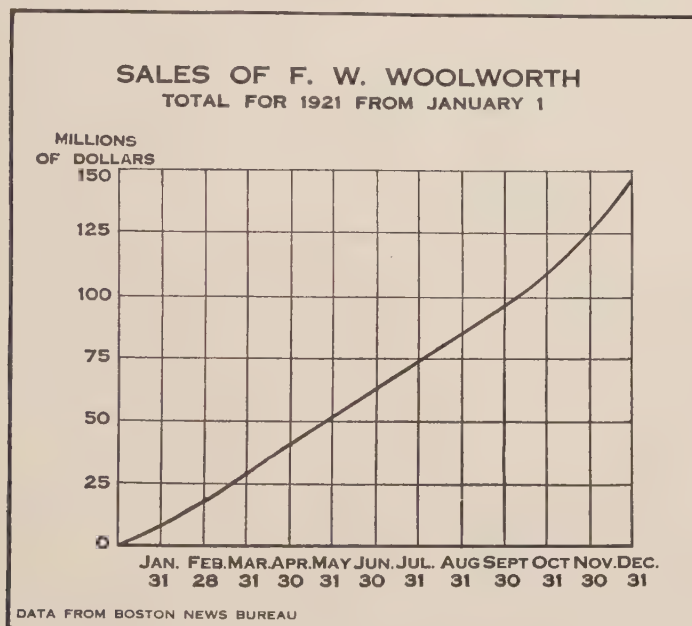


FIG. 126.—Smoothed curve. (Cumulative.)

For instance, the point halfway between Jan. 31 and Feb. 28, represents very closely the sales accumulated from the beginning of the year to the middle of February. In variables such as population figures (Fig. 127) changes rarely occur suddenly, and sharp angles nearly always would be absent from such a curve. So in plotting cumulative data or a continuous series the points should be connected by a smoothed curve as a rule, and not by straight lines. In smoothing a curve, the maximum possible radius of curvature should be maintained constantly, and all sharp breaks should be avoided unless such breaks are actually known to have occurred, as in Fig. 127 just before the year 1350.

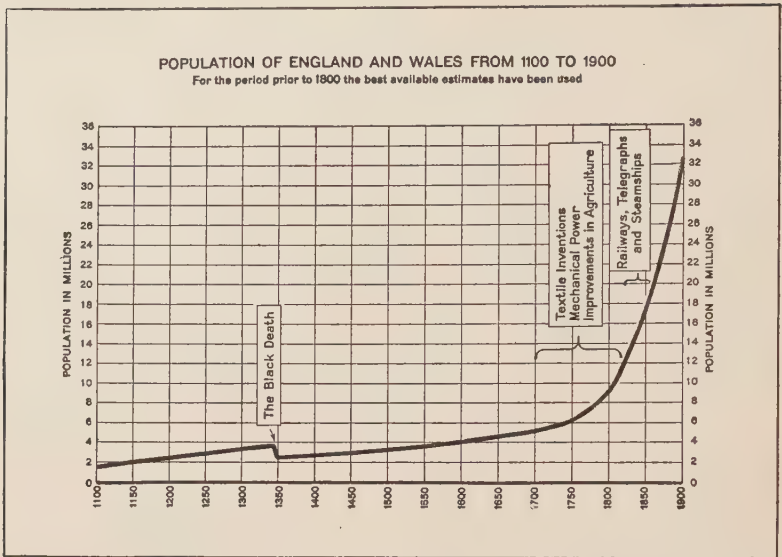


FIG. 127.—Both smoothed and broken curve. (An abrupt break occurs just before 1350. The remainder of the curve is smoothed and represents gradual changes.) (Courtesy of M. C. Rorty.)

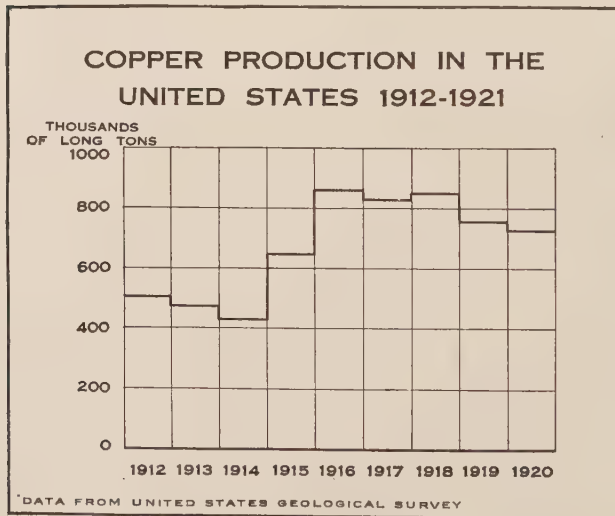


FIG. 128.—A step curve. (Usually a poor form.)

Throughout this discussion, curves like Fig. 125 will be referred to as broken curves, and those like Fig. 126 as smoothed curves. The subject of smoothing curves is discussed in greater detail on pages 162 to 164 inclusive.

Step Curves.—The step curve shown in Fig. 128 seldom is good practice. It does not show the direction of the change over a period as well as the broken curve, and it does not bring out relations at individual points as well as a bar chart. If two or

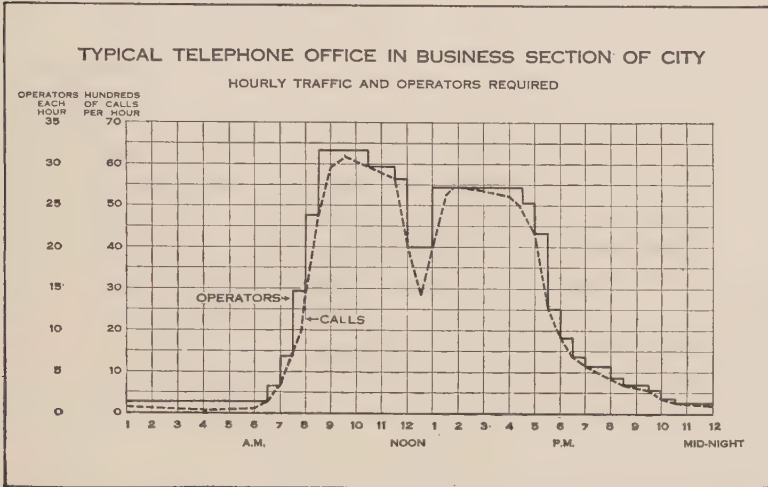


FIG. 129.—The step curve is correct for showing changes which occur abruptly as in the curve of operators in the above chart.

more step curves cross one another, they are very confusing as the vertical parts coincide. This form of curve should be used, however, if the change takes place abruptly, as in showing the number of operators in Fig. 129.

Polar Coordinates.—When the problem is one of presenting data of phenomena which are repeated regularly, as every year or every day, the form shown in Fig. 130 is sometimes used. While it may have the advantage of showing a continuous recurrence better than a curve plotted on rectangular coordinates (Fig. 129), its use should be discouraged in most instances, as the method used in Fig. 129 is much easier for the business man to read and understand than a chart which is drawn on the radial plan (polar coordinates) of Fig. 130. An exception to this rule may be made, however, in the case of showing a graph in relation to a clock or other dial instrument. For instance, an

effective chart based on the same general plan of Fig. 130 is shown in Fig. 131. This method of shading areas, however, exaggerates the figures.

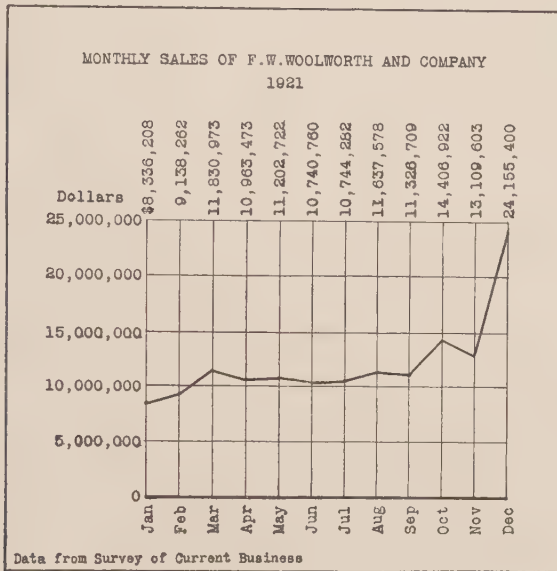


FIG. 132.—Tabulation of data on chart.

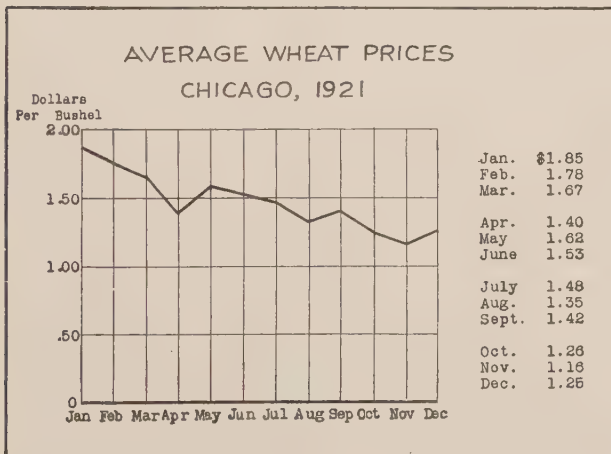


FIG. 133.—Tabulation of data on chart.

While occasionally this type of chart may be used effectively, it is not recommended for general use, as the reader is likely to

interpret the chart on the basis of area rather than lengths of radii. When the reader is familiar with their uses, however, polar coordinates are not objectionable as used on various kinds of recording devices, such as voltmeters, ammeters, steam gages, water pressure gages, and the like.

Presenting Tabular Data on Charts.—Usually it is desirable to present the numerical data along with the graph. This may be done as illustrated in Fig. 132 in which the data are included at the top of the diagram, or it may be done by presenting a separate table as part of the chart, as shown by Fig. 133. However, in Fig. 133 the table need not be placed at the end of the graph if it is more convenient to place it elsewhere on the chart. If the table is long or complicated, it is best to present it separately rather than as part of the chart.

COMPARISONS OF SIZE DIFFERENCES BETWEEN TIME VARIABLES

Probably the greatest use of graphic methods in business statistics is in showing not only how the size of a single variable changes, but in comparing the sizes of two or more variables over the same period.

Choice of Forms—Bars and Curves.—When, for several items, one point of time is to be compared with another, a bar chart like Fig. 134 is easier to read than a curve chart. In this case, eight different items are represented for 1919 and 1920. In order to make the comparison stand out clearly, all of the 1919 bars are shaded alike, and another kind of shading is used for the 1920 bars. Thus, one can easily compare the different items for each year simply by observing the same kind of shading or color throughout the comparison. When this kind of comparison is made, the bars should be arranged in groups for each item represented as shown in Fig. 134. The time scale for each group should read from left to right as shown. Sometimes the bars of charts like Fig. 134 are arranged in order of size, but this is not good practice if the normal order from left to right in the time scale is changed. This means, for instance, that in Fig. 134 the year 1920 should follow 1919 as shown, and not precede it for the sake of the order of size.

In Fig. 135 there is represented a slightly different problem. Figure 134 shows 8 items for 2 years, while Fig. 135 shows 2 items for 8 years. In Fig. 134 the different shadings represent

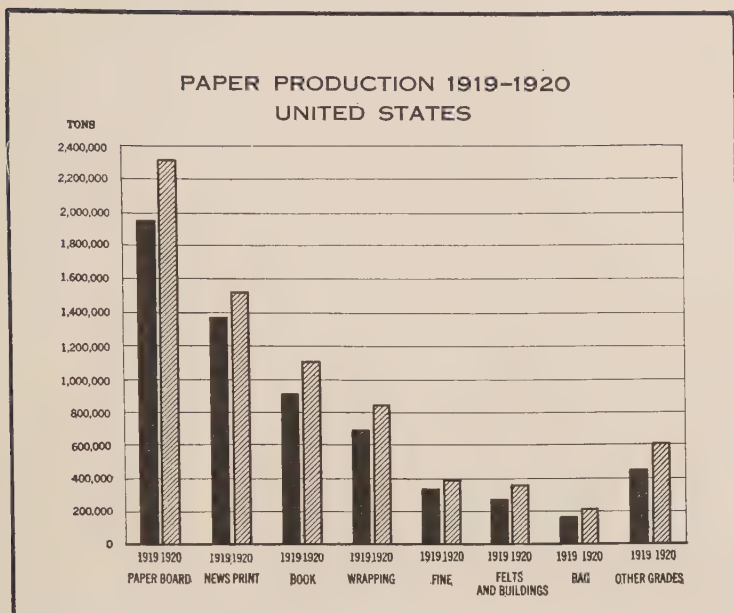


FIG. 134.—Two points of time compared for eight pairs of items. (Courtesy of News Print Service Bureau.)

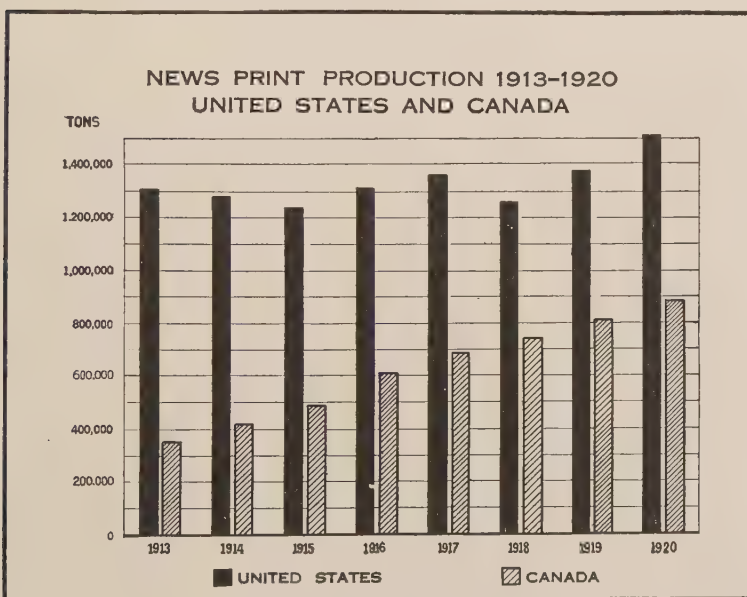


FIG. 135.—Two items compared at eight different points of time. (Courtesy of News Print Service Bureau.)

different *years*. In Fig. 135 the different shadings represent different *items*, that is, one kind of shading represents the United States over the period, and the other represents Canada.

Bar charts like Fig. 135 are not easy to read if more than two series are represented. It is confusing to follow three or more different kinds of shaded bars for each year over a period. For two variables the bar chart will show the comparison quite effectively but the curve method is most commonly used in cases of this kind. Figure 136 shows the data of the bar chart in Fig.

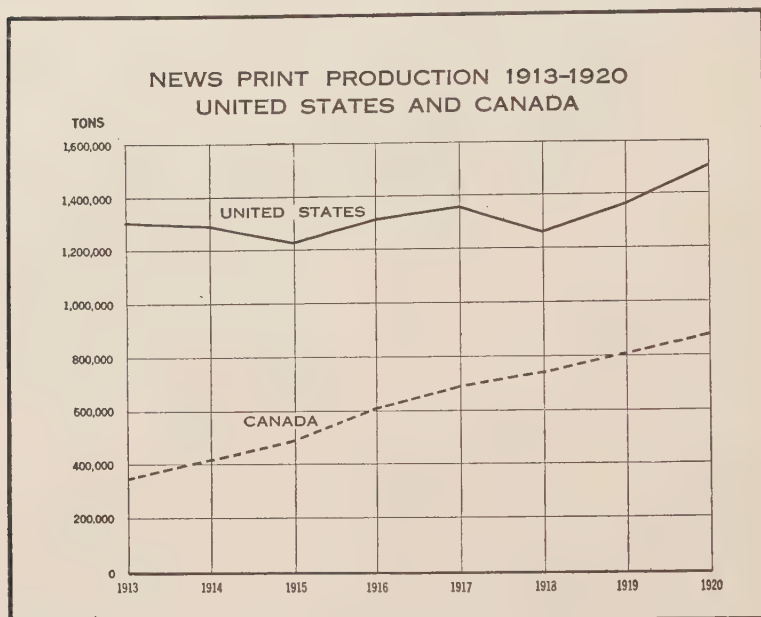


FIG. 136.—Data of Fig. 135 shown as curves. (The most common method of presenting this kind of comparison.) (Courtesy of News Print Service Bureau.)

135 plotted as arithmetic curves. Both charts are read in the same way and give the same impression.

Scales for Size Comparisons.—In Fig. 135 we compare the lengths of bars representing the United States with the lengths of those representing Canada. In Fig. 136 we compare the space between zero and the curve for the United States with the space between zero and the curve for Canada. This shows how important it is to begin with zero and to show the whole vertical scale. Thus, Fig. 137 (without the dotted addition) is likely to leave the visual impression that May futures are worth twice as

much as October futures over the period shown, while, as a matter of fact, by studying the scale one can see that May futures are worth only about one-tenth more. This true relation would be shown clearly if the chart were drawn to include the entire scale as indicated by the dotted lines. If for some reason the entire scale is not used, the reader should be warned as explained in connection with Fig. 116 on page 89.

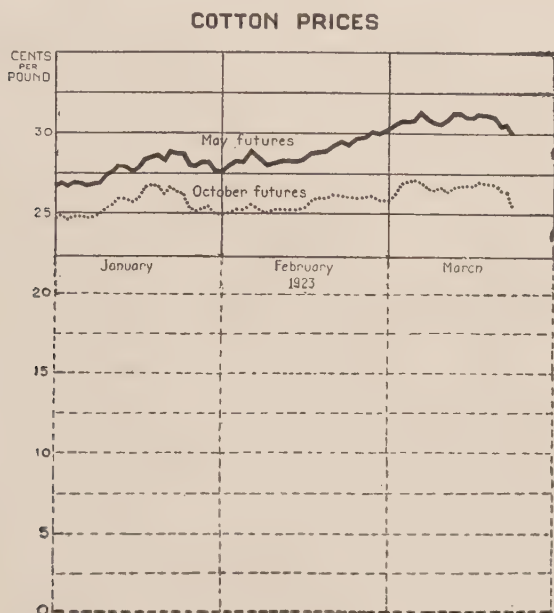


FIG. 137.—In comparing sizes it is important to include the whole scale.

Variables of positive and negative values may be compared by plotting above and below the zero line as shown in Fig. 138. Note that the shading of the resultant tonnage is especially effective.

When comparing sizes, it is necessary to have a scale that is common to all of the series considered. That is, we cannot compare cotton in bales with wool in pounds and get a true impression of the relation between the amounts of the two textiles. This can be gained only by using a common unit, say, pounds. We cannot directly compare quantities of oil in barrels with tons of coal, but, by reducing them to pounds, monetary values,

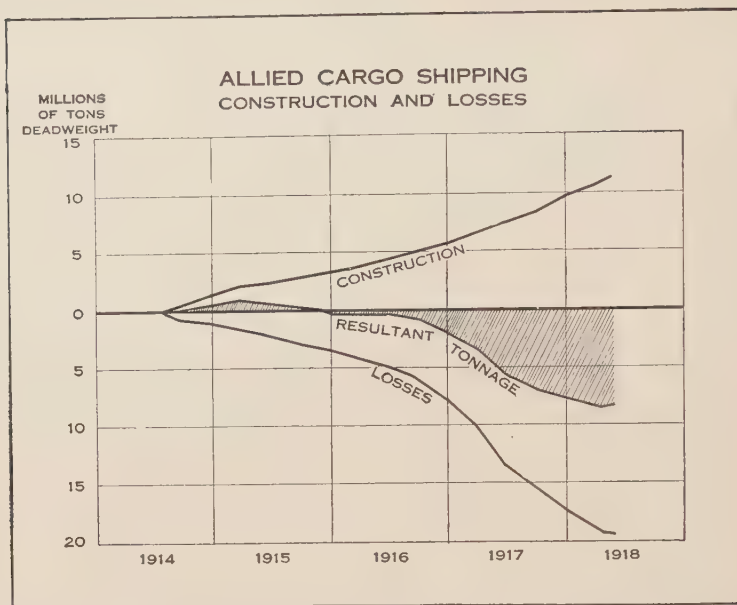


FIG. 138.—Comparison of positive and negative values. (Adapted from General Electric Co.)

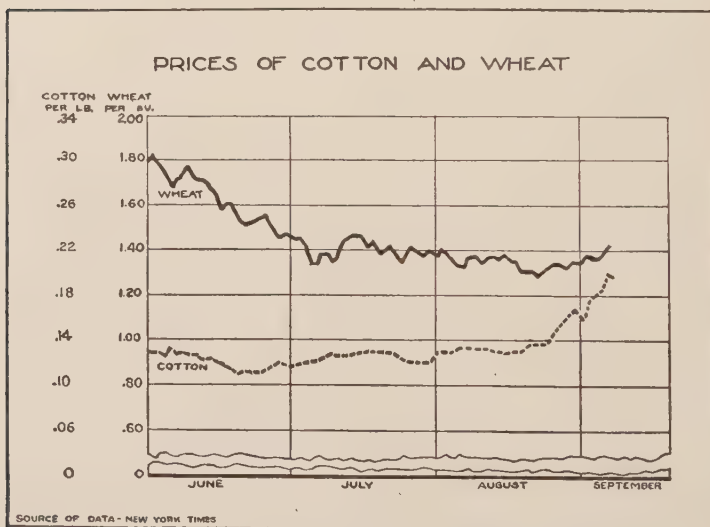


FIG. 139.—Not a size comparison. Usually bad practice.

heat units, or other units common to both, we can plot them to the same scale and make direct comparisons.

Occasionally a chart is constructed like Fig. 139, which is intended to show simply the fact that wheat and cotton prices have increased. It is not intended to show how much or at what rate the prices have gone up. In such a chart the scale may be cut off, but this fact must be shown, when it occurs, by a horizontal break across the graph, as illustrated in Fig. 116A, or by a broken line across the bottom, as in Fig. 116B. While in Fig. 139 there are to be no comparisons between the prices, it is dangerous practice to use two scales as illustrated, on arithmetic charts. In Fig. 129 on page 99 two different scales are used, and since there is a definite relation between the two, a good comparison can be made. Situations occur only occasionally, however, when two arithmetic scales of different units can be used correctly.

When two scales are used, both should be placed on the same side of the graph rather than one on each side. With a different scale on either side, one is likely to read the left-hand ends of the curves from the left-hand scale, and the right-hand ends from the right-hand scale. The double scale arithmetic chart, as used in Fig. 139, is not recommended for general use, as it is loose in its construction and is likely to mislead the reader who may assume that the curves may be compared directly, since they are placed on the same chart. It would be better to draw them to a common scale or on separate charts. If the *sizes* of the items are to be compared over the time period, it is imperative to have a common arithmetic scale. If *rates of change* are to be shown, the curve should be plotted as described in the following chapter. In nearly every case it is dangerous to use more than one arithmetic scale on a graph.

Differentiating Curves.—When two or more variables are represented on a graph, it is necessary to so differentiate the curves that they will not be confused. This is the best practice, even if the curves are in different parts of the field and do not cross, in which case there would be no confusion if they are alike. If they are close together and occasionally cross one another, each curve must be made in such a way that it can be followed without confusion. There are two common methods of differentiating curves. One is by using different colors, and the other is by using different kinds of dot and dash combinations in black

such as those used in Fig. 140. The latter method is the one usually used when the charts are to be reproduced by any kind of a photographic process.

Designating Curves.—When two or more different curves are used, it is necessary to designate them with the names of what they represent. This is done in two different ways. One is to place the names along the curves near the left-hand ends or beginnings

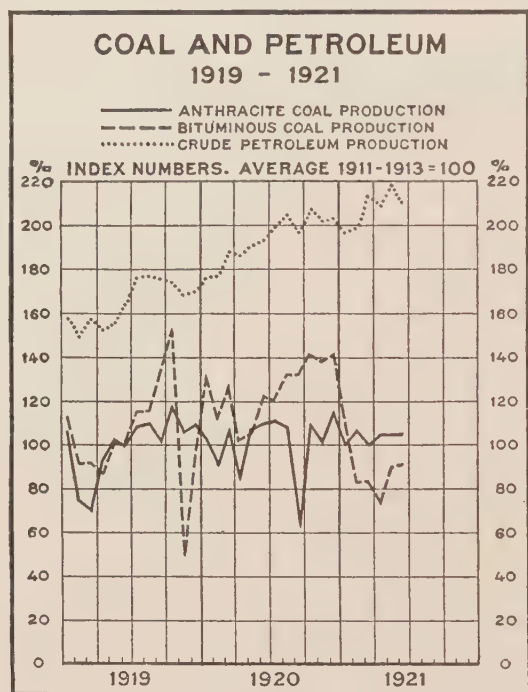


FIG. 140.—Differentiating and designating curves. (Courtesy of Federal Reserve Bulletin.)

of the curves (Fig. 141). This is the most convenient from the point of view of the reader, since the designation is directly connected with the curve. The other method is to designate the meanings of the different kinds of lines by a separate key or legend elsewhere on the chart as in Fig. 140. Of these two methods, the one illustrated in Fig. 141 is recommended as best for general use.

Number of Curves.—The number of curves that can be placed successfully upon a single graph depends chiefly upon the way in

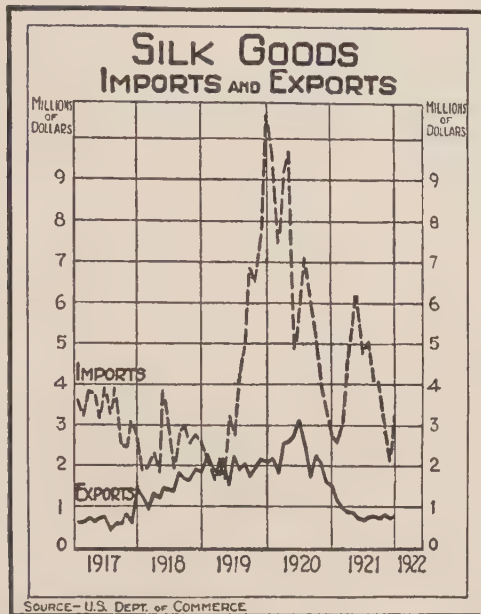


FIG. 141.—Designating curves. (Best method.) (Courtesy of Federal Reserve Bank of Philadelphia.)

PRODUCTION OF BITUMINOUS COAL IN MICHIGAN AND STATES WEST OF MISSISSIPPI RIVER
1917 - 1918

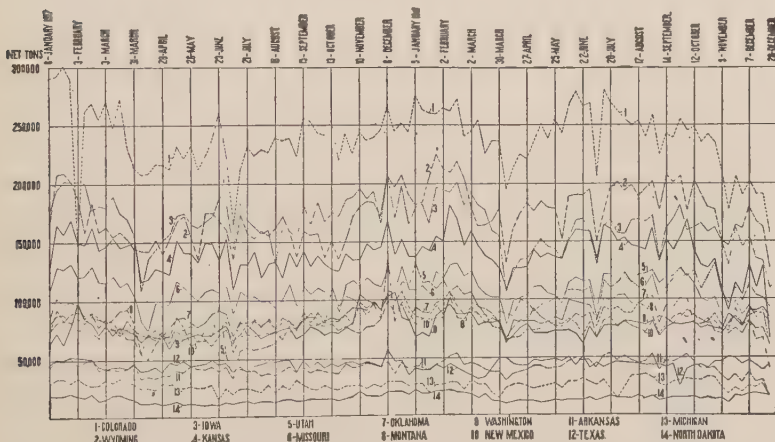


FIG. 142.—Hopeless confusion. Too many curves. (Other errors are light zero line, and wrong position of time scale.)

which they are distributed over the field, but in no case should as many curves be plotted on one chart as are shown in Fig. 142. Such a chart is useless as it is almost impossible to follow a curve across the chart. In Fig. 143 five curves are too many as they run together so badly that they can be distinguished only with difficulty. An exception may be made to this rule, however, when curves are plotted together to show correlation. Note also that in Fig. 143 a change was made in the horizontal scale. When this is done, there should be a distinct break in the graph where the change occurs. In this chart it would be better to

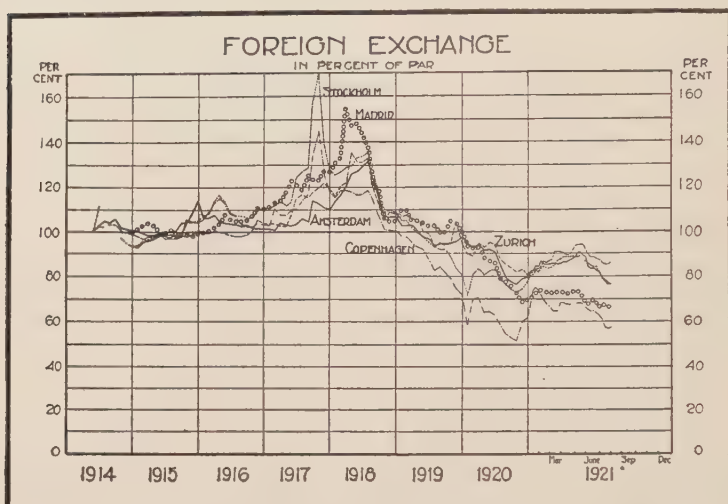


FIG. 143.—Confusion of curves. (Also incorrect time scale.)

give 1921 the same space as the other years and plot the data accordingly.

Thirteen curves are successfully shown in Fig. 144. A large number of curves may be shown clearly and compared easily by using this method of dividing a chart into a number of small graphs which, if plotted as a single graph, would result in hopeless confusion. When charted as in Fig. 144, all of the small graphs should be drawn to exactly the same scales. Otherwise, they cannot be compared easily and they probably will be compared inaccurately.

Standardization.—Annual figures or data for other definite periods are often placed on different charts for convenience, and when it is desired to compare these different charts it is imperative

RESERVE PERCENTAGES OF FEDERAL RESERVE BANKS

DISREGARDING THE CREDITS EXTENDED BY VARIOUS
FEDERAL RESERVE BANKS TO OTHER FEDERAL RESERVE BANKS

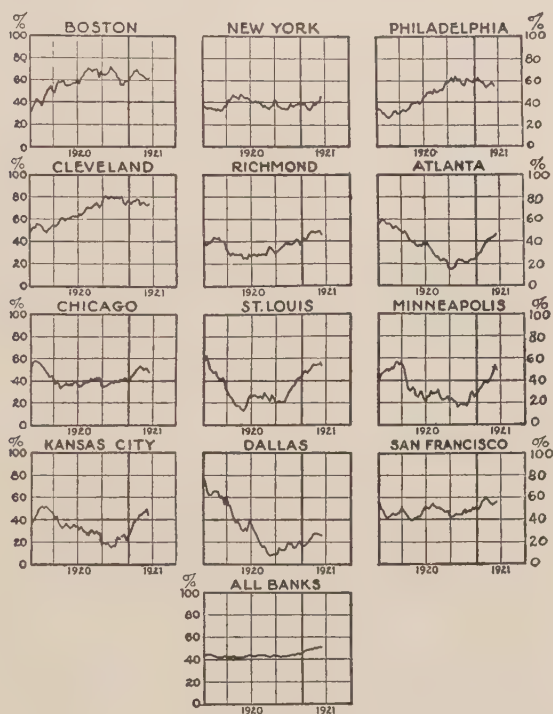


FIG. 144.—A good method of showing a large number of curves. (Adapted from *Federal Reserve Bank of New York.*)

to have them drawn to the same scales. That is, the units of the vertical scales must be the same on all the charts and the units of the horizontal scales must be the same on all the charts. For

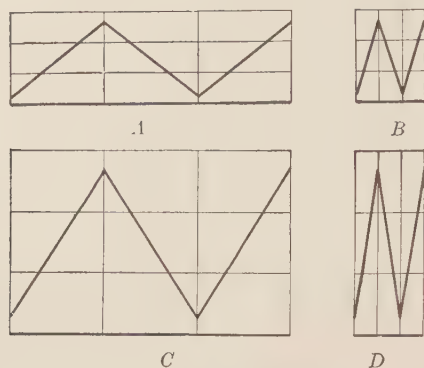


FIG. 145.—Distortion caused by different scales.

instance, if 1 inch equals \$100 in the vertical scale of one of the charts, it should equal the same amount in the other charts, and if 1 inch equals 1 year in the horizontal scale of one chart, it

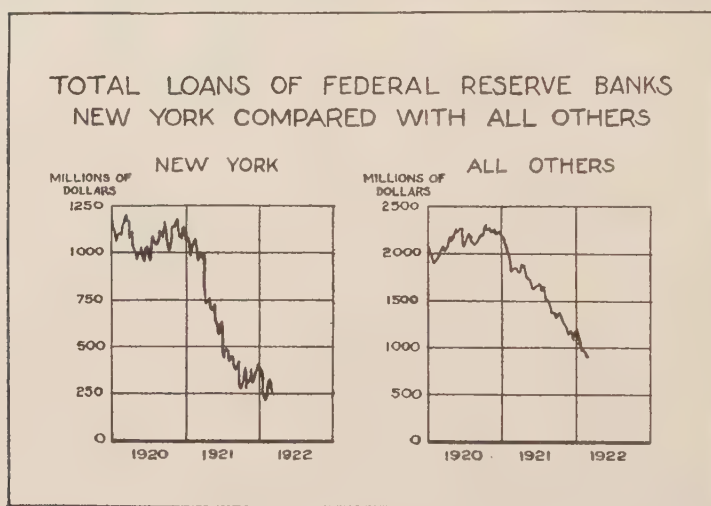


FIG. 146.—Juggling of scales. Usually a dangerous practice.

should equal the same period in the others. Without this standardization, the curves may be distorted as in Fig. 145, which shows how exactly the same data appears when plotted to different kinds of coordinate spaces. Thus, in comparing sizes over a

period of time, a common scale must be used whether or not the comparisons are made by curves on the same or separate charts.

In Fig. 146 arithmetic curves have been juggled in an attempt to show relative change. That is, the range of the variation is given about the same space in the vertical scale of each graph, and the units are laid out accordingly, which in this figure is in a ratio of one to two. Then, if the "New York" curve drops from 1,000 to 500, it is a decrease of 50 per cent. If the "All Others" curve drops from 2,000 to 1,000, it also decreases 50 per cent. These changes would be represented by the same distances on the scale when they are arranged as shown in this figure. This

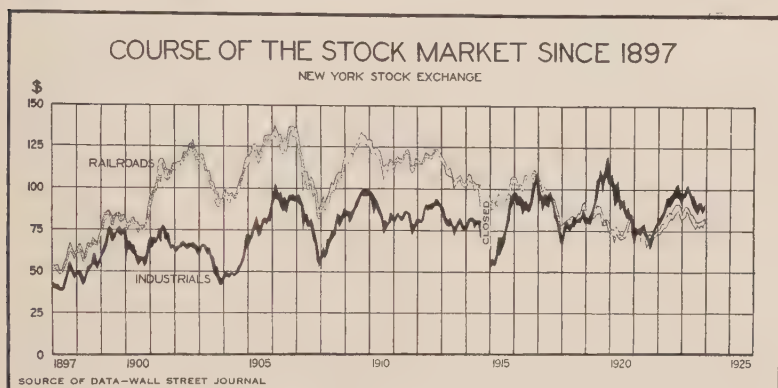


FIG. 147.—Range of variation. (Shading between curves.)

practice, however, is not advised, as it is best to use the logarithmic chart when showing rates of change.¹

Range of Variations.—In Fig. 147, curves are plotted for both high and low prices and the area between is shaded. Thus, the range of stock prices over the period is clearly seen. Another good method of representing range of prices is shown by Fig. 148. Here, the top of the bar represents high and the bottom represents low, while the length shows the range, and by their position on the vertical scale the ranges for different dates can be compared.

Double Time Scales.—Figure 149 illustrates the use of a double time scale in the comparison of two separate periods. Here the units are not different, as yearly intervals are used for

¹ For further explanation of the dangers involved in this practice, see the discussion of Figs. 185 and 186.

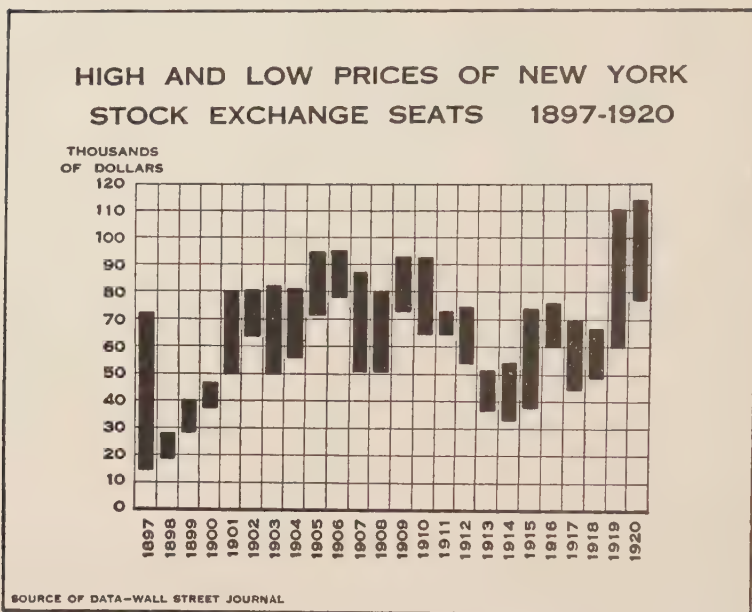


FIG. 148.—Range of variation. (Bars.)

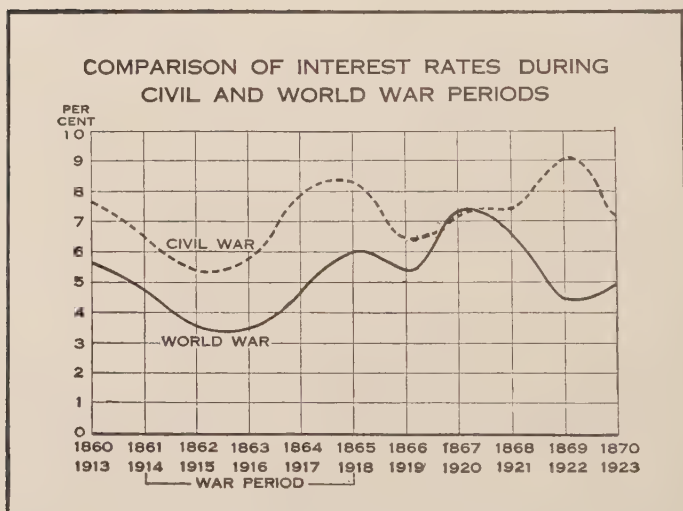


FIG. 149.—Comparison of different time periods.

both curves. Simply two different sections of time are compared. In this way, one year in months or weeks is often compared with another in studying seasonal variations. These curves are freely smoothed from the points plotted on the vertical lines.

Different Time Scales.—Often it is desirable to show the more current data in greater detail than the more remote. Thus, in Fig. 150 the current year is shown on a monthly basis while the preceding years are shown on an annual basis and two different time scales are used. When data are shown in this way, it is very important for the sake of clearness and accuracy to leave a substantial space between the two graphs as shown in Fig. 150. Sometimes in common practice they are placed together but this is confusing and often results in wrong impressions. This particular chart (Fig. 150) is taken from a monthly report in which the same form is used each month. The original form is laid out at the beginning of the year and with each month it is brought up to date and a photostat copy made of it.

Smoothed and Broken Curves in Comparisons.—The rule for choosing between smoothed and broken curves in comparisons is, of course, the same as previously discussed for the single curve. If the value represented for each period does not include any of the value for the previous period, all that the curve shows is the direction of the difference in the new figure, that is, whether or not it is greater or less, or the same. The values are represented by the plotted points and there is no value on the curve between points. Thus, in the case of the monthly sales curve in Fig. 151 (solid line) the point plotted on the January ordinate represents the sales for January, the point plotted on the February ordinate represents the sales for February, and so on. The ordinate represents the month, and the space between the lines does not represent any more than does the space between bars when comparing, say, populations of New York and Chicago. The only purpose of the curve in showing non-cumulative data over a period of time is to indicate the direction from one point to the next, and to define the area between the points and the zero line, which area is used as a basis in comparing the different series.

On the other hand, if the value at a point includes the value represented in the preceding point and is merely a change in the total, or if the new value is simply a variation in the old value, there seldom will be abrupt breaks, and a smoothed curve should be used in most instances. Thus, the smoothed curve was used

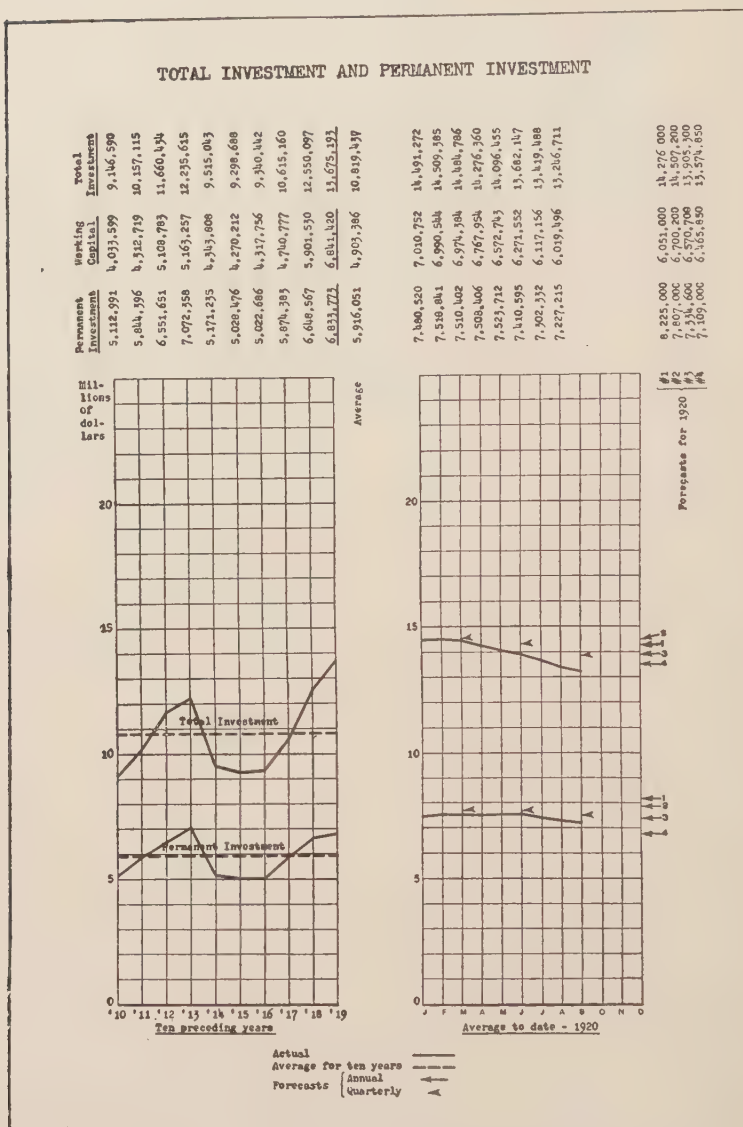


FIG. 150.—Different time scales. Separate graphs for showing remote and current data together on one chart. (Courtesy of E. I. du Pont de Nemours & Co. Data fictitious.)

in Fig. 149 because the old rate changes—there is not an entirely new rate established each year. In Fig. 151 the curve for “Total since first of year” (dotted line) is cumulative and is gradually changing throughout the space between the plotted points. Thus, the point on the dotted line midway between the lines representing June 30 and July 31 would give, very closely, the total accumulated from the first of the year to July 15.

When curves are smoothed as freely as those shown in Fig. 149, they are sometimes misleading to those who do not understand them. In common practice, such curves are often plotted as broken curves.

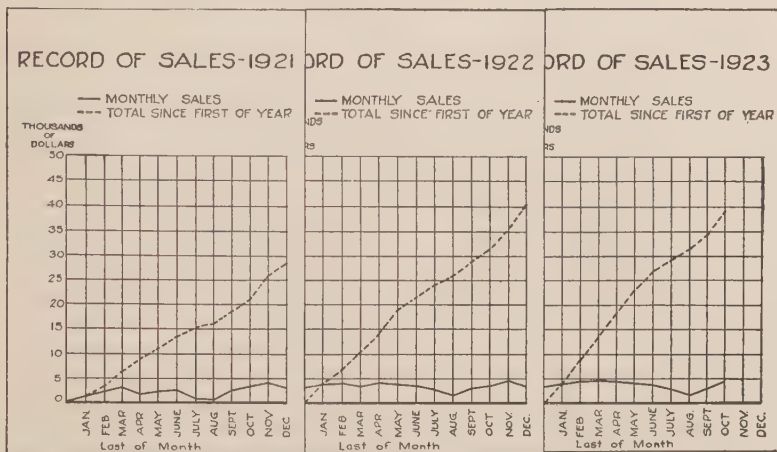


FIG. 151.—Use of separate cards for showing continued series.

Since in cumulative figures a period of time is represented by the space between the designated dates, it is necessary to have as many *spaces* as there are plotted points in charts such as Fig. 152. In this chart the figures are plotted on the last of each month. In order to have the chart complete, a space must be allowed at the beginning of the graph to show the accumulation during the month of January. In annual graphs of non-cumulative figures, twelve lines (eleven spaces) are all that are needed if each line represents a month, but when such charts are drawn to be compared with other charts, as in Fig. 151, it is well to have an extra space at the beginning or end of non-cumulative graphs in order that the space between January and December will be the same as between the other months when the graphs are placed end to end.

Shading to Show Differences.—The plan of shading areas, as shown in Fig. 152, often can be used to bring out certain facts more vividly than when curves alone are used. In Fig. 152

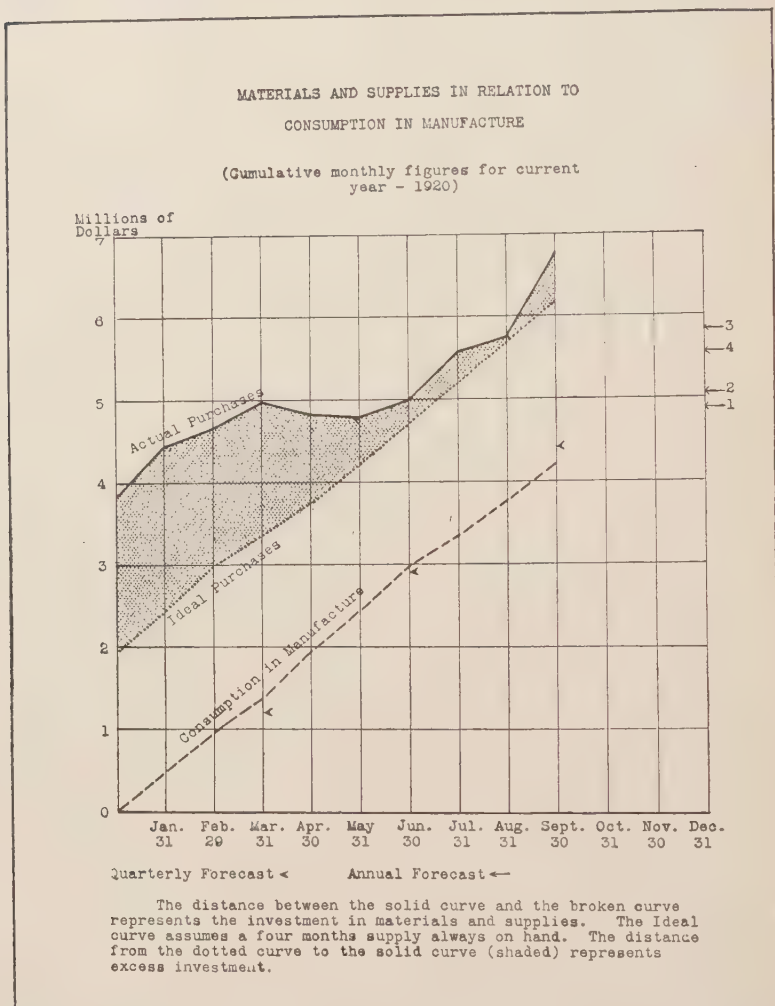


Fig. 152.—Shading to emphasize differences. (Adapted from E. I. du Pont de Nemours & Co. Data fictitious.)

the dashed line represents the materials consumed in manufacture. The dotted line represents the figures at which the raw material must stand if there is always to be a 4 months' supply on hand. Note that the dashed line reaches the dotted line values

just 4 months later. The solid line represents the actual inventory on hand. In order to emphasize the difference between the actual and ideal supplies, the area between the two lines was shaded, and this brings out very clearly the excess of investment in raw materials.

A similar method is used in Fig. 153, in which the solid line represents the ideal expense budget of \$100 per month. The dashed line shows the actual expenditure to date and how much

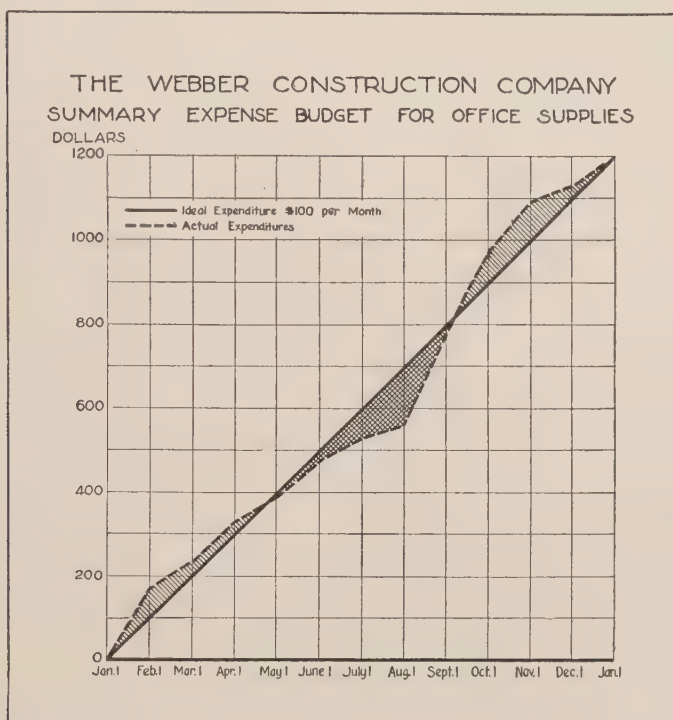


FIG. 153.—Shading to emphasize differences between ideal and actual expense.

it is above or below the ideal. In the original of this chart the areas above the solid line were shaded in red and those below in black. This method of shading is very effective in calling attention to whether the expenditures are too large or too small.

Indicating Forecasts.—The arrows on the chart in Fig. 152 represent forecasts. At the beginning of the year a forecast is made for the manufactures curve for 3 months and this point is indicated by the arrowhead at March 31. At the same time the

point which the curve will reach at the end of the year is estimated, and indicated by arrow number 1 at the end of the graph. The quarterly forecasts are made at the end of each quarter, and at the same time the forecast for the end of the year is revised and indicated as shown. Other forecasts, such as a moving annual forecast, may be indicated in the same way.

Another method of indicating forecasts is to use separate curves.

COMPONENTS OF A TIME VARIABLE

Choice of Forms—Bars and Curves.—Often it is desirable to show the component parts of a time variable and this may be done in various ways. Figure 154 illustrates a simple method of

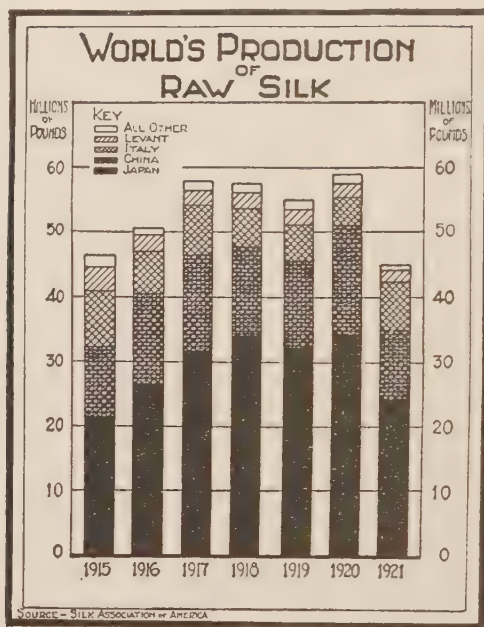


FIG. 154.—Component parts of a time variable. (Separate component bars.)
(Courtesy of Federal Reserve Bank of Philadelphia.)

showing how the world's silk production is divided over the period from 1915 to 1921. This chart shows very well the distribution for any one year but it is not as easy to see the distribution over the whole period. The lines connecting the components of the bars in Fig. 155 are intended to aid in following a certain component over the period, but this chart is hard to read, as the

cross-hatching is not distinct and there are too many small components to allow clean-cut comparisons. In Fig. 156 the chart is easy to read, as the components are continuous over the period and can be followed directly across the chart. This is possible, however, since the data do not change much, and the same kinds of components are always opposite each other. That is, for instance, in Fig. 156 an "Other States" component is never opposite a "Balance Imported" component. This difficulty

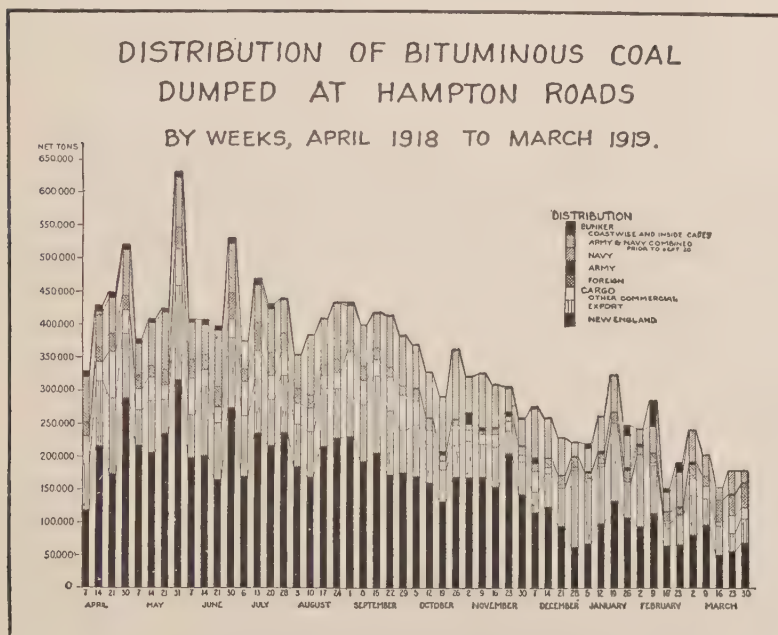


FIG. 155.—Component bars connected by lines.

often happens in Fig. 155, which would be easier to follow over the period if plotted in curves and the areas filled in between as in Fig. 157. Figure 156 should be considered as bars placed together—not as step curves with the areas filled in between them. The reason for this can be seen by referring to Fig. 155, where the same kind of component may, at a given date, be far above or below its position on other dates. Figures 157 and 158 give a more continuous picture of the series than do the bar charts in Figs. 154, 155, and 156.

Designating Shaded Areas.—Two methods of designating the "belts," as the areas between the curves are called, are

illustrated by Figs. 157 and 158. If there is room for the designations, the method of placing them on the belts, rather than in an outside legend or key, makes them easier to read. Large white spaces should not be blocked out in the belts for the designations, as this interferes with the conception of the sizes of the areas.

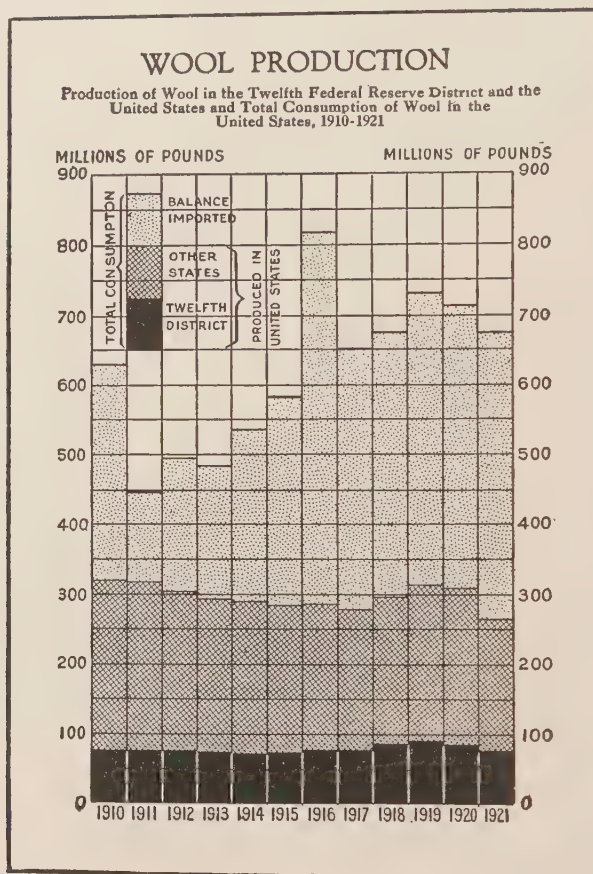


FIG. 156.—Component bars with no intervening spaces. (Courtesy of Federal Reserve Bank of San Francisco.)

Actual Sizes and Percentages.—So far, the discussion of component parts of time variables has considered only actual sizes of the values. Frequently it is desired to know what percentage changes have taken place without regard for the actual sizes of the changes. In instances where the total does not change in size over a period, the actual size distribution and the percentage

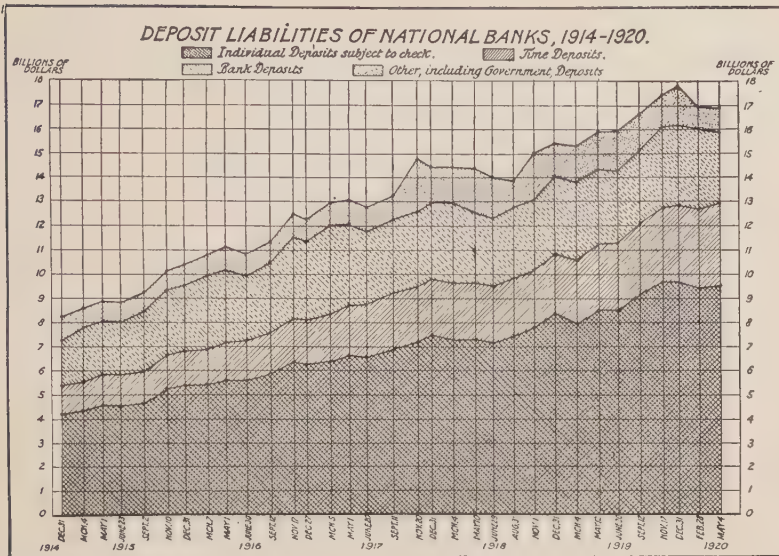


FIG. 157.—Shading of areas between curves to show component parts. (Courtesy of Federal Reserve Board.)

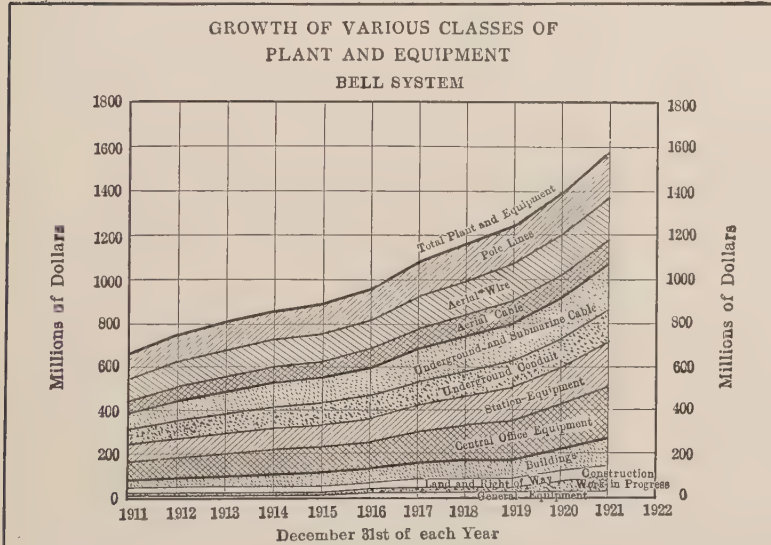


FIG. 158.—Designation of belts on a component time chart. Compare with legend on chart in Fig. 157. (Courtesy of American Telephone and Telegraph Company.)

distribution of the components would appear the same when shown graphically. When the totals vary in size over the period, they



Fig. 159.—A simple method of showing time changes within the whole. (Courtesy of the National Industrial Conference Board.)

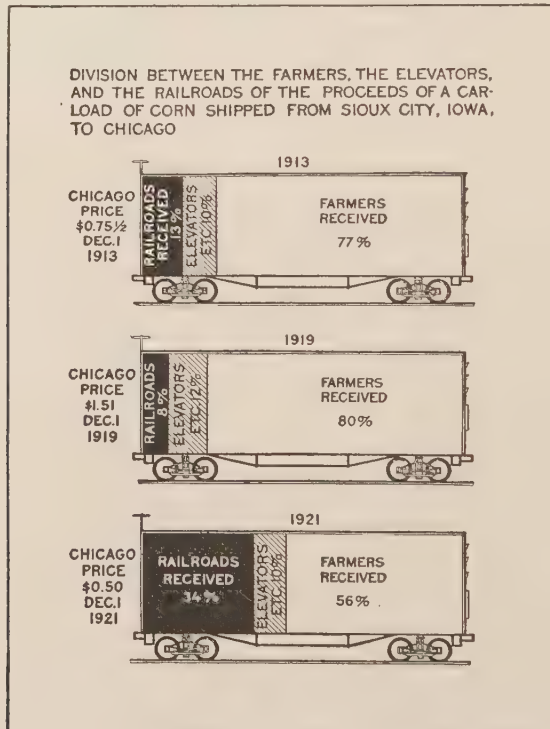


Fig. 160.—A pictorial component time chart. (Courtesy of the United States Department of Agriculture.)

may be represented equally as 100 per cent, and the components plotted proportionally.

In Fig. 159, each circle represents 100 per cent and the sectors represent the components. This chart must be read carefully,

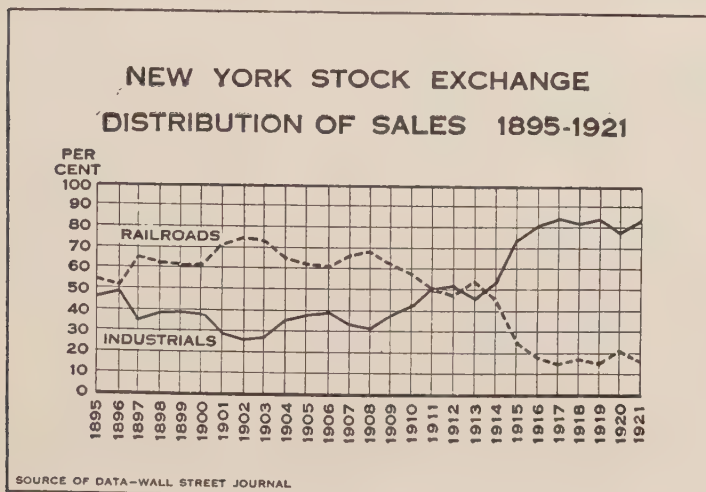


Fig. 161.—A method of showing percentage relations of two components.
(Difficult to read.)

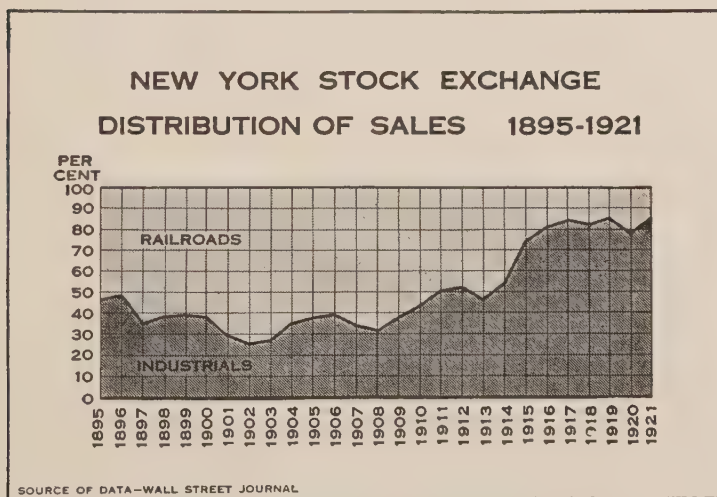


Fig. 162.—A much clearer method of presenting the data of Fig. 161.

however, as the circles are equally spaced but equal spaces do not represent equal intervals of time. Otherwise, it is a clear and

simple presentation which would be understood readily in popular publications.

Another simple chart that is understood easily is shown in Fig. 160. Pictorial charts, however, are valuable only for popular presentations.

It is easier for the average reader to grasp the relative values of Fig. 162 than of Fig. 161. A conception of the relations in Fig.

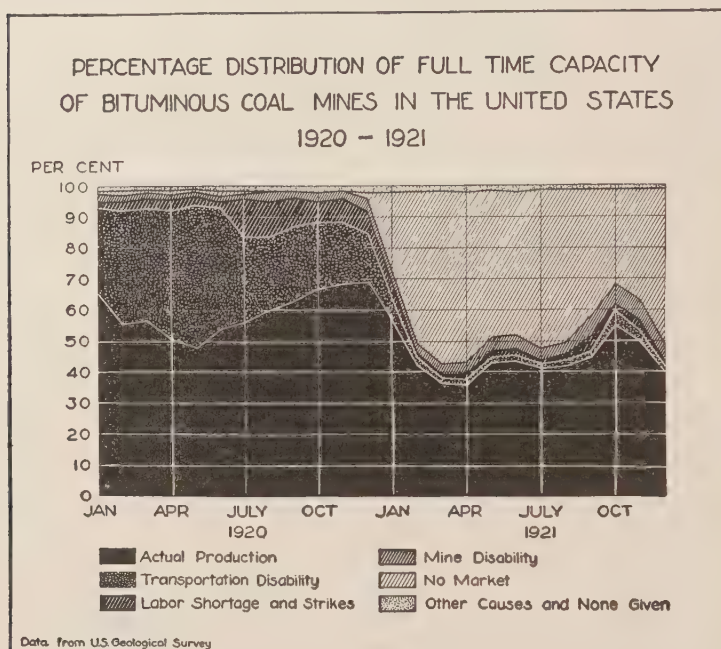


FIG. 163.—Use of component curves to show percentage distribution of several components.

161 can be gained with less effort if but one curve is used and the areas are shaded as in Fig. 162. The shading gives the components a sharp contrast, and makes the chart clear to a large number of persons who ordinarily would be confused by the chart made by using curves as shown in Fig. 161.

When several widely varying components are to be shown, it is much easier for the eye to carry them across the period if they are presented in the belt chart form (Fig. 163), than when they are presented in the bar form as shown in Fig. 164. However, the comparisons of the irregular areas in Fig. 163 are not

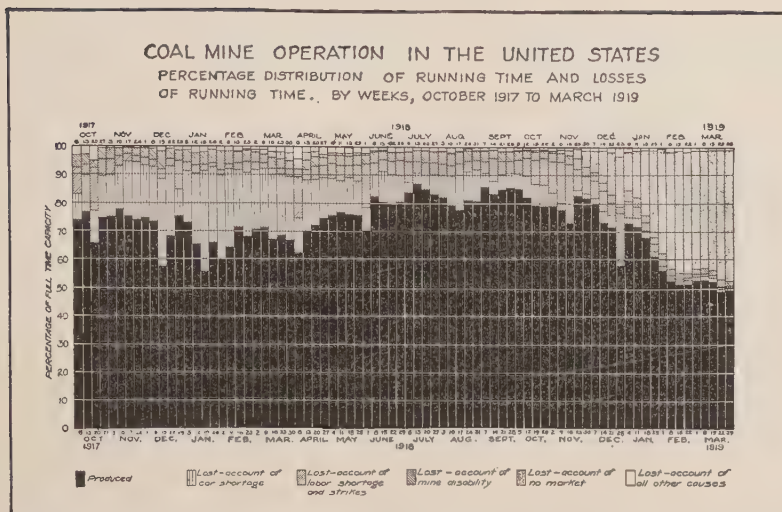


FIG. 164.—Use of component bars to show percentage distribution of several components. (Courtesy of the United States Geological Survey.)

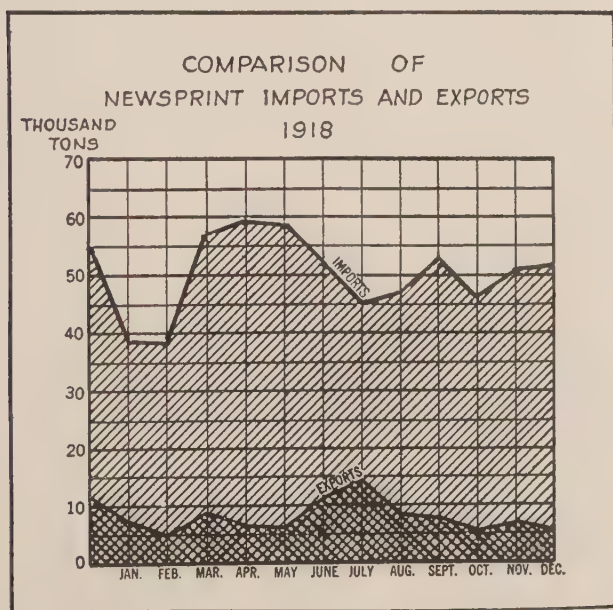


FIG. 165.—A common error in use of shading. Not component parts. Both curves are measured from the zero line.

easy to make and care must be taken not to compare the widths of the belts. Either the areas or vertical distances between the curves must be used as the bases for comparisons.

A Common Error.—When the areas between curves are shaded, they must not be assumed to overlap. Such shading (Fig. 165), is misleading. The chart in Fig. 165 is wrong as the lower curve represents an entirely different series and not a component part of the top curve. The curve of imports is measured from the zero line and not from exports as a base. Exports are not a part of imports and hence the areas should not be shaded.

Shading Components.—The shading for the different divisions of a component time chart must be chosen very carefully if the graph is to be easy to read. Colors can be used very effectively in component charts and, when used, should be representative, as far as possible, of the thing shown. For instance, the original of Fig. 163 was a colored wall chart on which black represented coal production, and blue represented business depression or no market. This point, however, in the use of colors is only of minor importance and cannot be carried very far. Black and white shading usually is arranged in order from dark to light, but sometimes this order is changed to get a greater contrast between the different components.

Checking List for Curves.—The following list includes the most important points to be observed in making the curve charts described in Chaps. V, VI, VII and VIII, and it can be used when checking arithmetic and logarithmic time charts, frequency charts, and correlation charts to see if they are complete:

Have you placed your vertical scale (dependent variable) on the left of the graph, and does it read from bottom to top? (If desirable to show scale on right, it may be placed there in addition to the scale on left.)

Have you placed your horizontal scale (independent variable) along the bottom of the graph, and does it read from left to right? (If desirable to place scale at top, it may be placed there in addition to scale at bottom.)

Have you centered a caption over the vertical scale?

If necessary, have you placed a designation under the horizontal scale?

If you have used more than one curve, have you designated them and can they be distinguished clearly?

If you have shaded or colored areas, have you included designations or a key or legend?

Are your curves heavy?

Is the zero line heavy?

Is your background of coordinate lines light?

Is your title your largest lettering?

Is the title simple and clear?

Have you given the reference or source?

Did you check for accuracy?

Have you ruled in a border line?

Have you given your name (lower right) and the date (lower left) if desirable?

Are pencil lines erased?

Will your chart be clear to the reader?

CHAPTER VI

HISTORICAL TIME SERIES (*Continued*)

So far in the study of time series we have considered only variations in *size* over the period and the charts were constructed on simple arithmetic scales. It is the purpose of this chapter to describe the construction and use of charts showing *rates* of change. This involves the use of the logarithmic scale. The purposes of arithmetic curves and logarithmic curves are entirely distinct. One cannot be used instead of the other and, in fact, a clear understanding of the arithmetic curve is practically impossible unless one understands also the logarithmic curve. While in the past there has been a good deal of prejudice on the part of the business man against the logarithmic chart, he is now using it widely since he has discovered its importance and realized its simplicity.

RATES OF CHANGE IN A TIME VARIABLE

The preceding chapter considered principally the different *sizes* in time variables as, for instance, in part A of Fig. 166 in which the size of the population for any one point of time can be compared with the size for any other date. That is, we can see that the population of 1920, for instance, is about twice that of 1880. We cannot, however, compare the rates of increase of one part of the curve with another. While the curve is steeper from 1910 to 1920 than it is from 1810 to 1820, it does not mean that population is increasing more rapidly in the later period. It means simply that the actual difference in *size* is greater. If the *rate of change* is to be shown, the logarithmic¹ vertical scale can

¹ The Briggs or Common system of logarithms, which is the system in ordinary use, expresses any given number as a certain power of ten. That is, the logarithm of a given number indicates what power of 10 that number is. Thus the logarithm of 10 is 1; the logarithm of 100 or 10^2 is 2; the logarithm of 1000 or 10^3 is 3, and so on. If the number is not an even power of 10, the logarithm is computed in the form of a fractional exponent, and thus the logarithms of any assignable number may be approximately expressed. The purpose of logarithms is to facilitate and shorten calculations involving multiplication, division, and the finding of roots and powers.

be used, as in part *B* of Fig. 166 in which the slope of the curve indicates the *rate of growth*. It is readily seen that this *rate* is falling off slightly since the curve is less steep in the later part of the period.

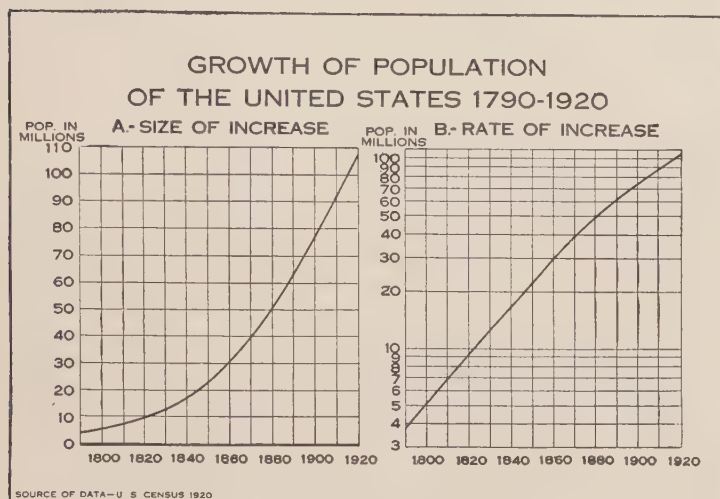


FIG. 166.—Arithmetic and logarithmic scales.

Simplicity of the Logarithmic Curve.—It must be clearly understood at the outset that the two parts of the chart in Fig. 166 have two different and distinct purposes. Part *A* shows *differences in size* but does not show the *rates of change*; part *B* shows the *rates of change* but does not show the *differences in size*. Both charts are equally simple, but, because this fact is not fully realized, the arithmetic scale (*A*) has been used more widely in charting business statistics than the logarithmic scale (*B*). Most business men regard anything that has to do with logarithms as something highly technical and extremely complicated, but, as a matter of fact, in order to make and use these charts it is quite unnecessary to have even a concept of logarithms. In fact, it is sometimes easier to explain the use of these charts to a man who has never heard of logarithms, than it is to make the explanation to a college man who has learned to regard logarithms as some species of intangible black art.

Scales for Logarithmic Curves.—Referring again to part *B* of Fig. 166 it will be seen that the horizontal scale is ruled arith-

metically and only the vertical scale is ruled logarithmically. Curves made in this combination are commonly referred to by business men as logarithmic curves.¹ The arithmetic horizontal scale allows the uniform spacing for the periods of time, such as

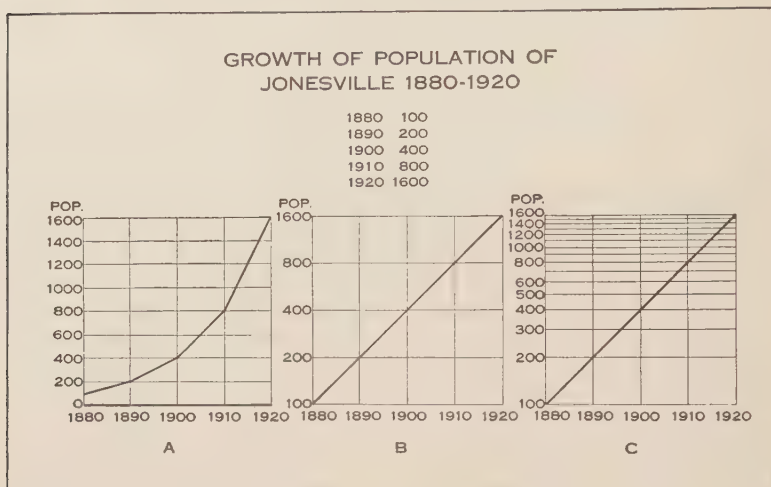


FIG. 167.—Scales in arithmetic and geometric progression.

months or years. The vertical logarithmic scale, which is in geometrical progression,² gives the true relative comparisons between the changes.

On the arithmetic chart, equal intervals on the vertical scale represent equal *differences*. Thus, in part A of Fig. 167 each

¹ In business statistics, this chart is commonly called a logarithmic chart or a logarithmic curve. Some writers suggest that it should be called a semilogarithmic chart, since only one scale is logarithmic, leaving the term logarithmic to apply to a chart in which both scales are logarithmic. Two logarithmic scales are so seldom used on a statistical chart, however, that the business user prefers to attach the modifier to this chart calling it a double logarithmic chart, applying the less awkward term "logarithmic" to the single logarithmic scale chart which is in common business use. The terms "semilogarithmic" and "arithlog" are used to describe some of the paper that is on the market for making these charts. Another term which is sometimes used is "ratio chart." (See Professor IRVING FISHER in "The Ratio Chart for Plotting Statistics," *Quarterly Publications of the American Statistical Association*, June, 1917.)

² A geometric progression is a series of numbers in which every successive term is multiplied or divided by a constant factor, thus 1, 2, 4, 8, 16, 32, etc., or 16, 8, 4, 2, etc. An arithmetic progression is a series of numbers in which every successive term is increased or decreased by a constant amount, as 1, 2, 3, 4, etc., or 2, 4, 6, 8, etc., or 15, 12, 9, 6, etc.

vertical spacing represents 200 persons and the curve shows the actual differences for each 10 years. In the logarithmic chart equal intervals on the vertical scale represent equal ratios. Thus, in part *B* of Fig. 167, each vertical spacing represents twice the preceding, or, in other words, each twofold or 100 per cent increase has the same spacing. In the table (Fig. 167), the increase from 1880 to 1890 represents a difference of 100 persons, and the increase from 1910 to 1920 a difference of 800 persons; these are the facts that are shown by part *A* of Fig. 167. However, the rate of increase is the same if 100 increases to 200 and 800 increases to 1,600; hence, there is no change in the slope of the curve in part *B* of Fig. 167 which shows these rates of change.

A logarithmic chart like part *B* of Fig. 167 would not be convenient for plotting between the ruled lines because there are such large intervals of value represented in the upper parts of the scales. A chart is much easier to plot and read if constructed on a basis of tens. Part *C* of Fig. 167 represents part *B* with horizontal lines interpolated at their proper places. In ascending order the lines are spaced closer together for, if there is to be a line for each 100, there will be none between 100 and 200 (Fig. 167*C*); one between 200 and 400; three between 400 and 800; and so on. Note also that these interpolated lines are not equally spaced but are always nearer the line above than the one below.¹

Semilogarithmic Paper and Ruling.—When it is desired to use the logarithmic curve, it may be made on special coordinate paper,² which is ruled as in Fig. 168, or the scale may be taken from the statistician's scale or from the slide rule.³ One

¹ The divisions of a logarithmic scale are proportional, not to the numbers themselves, but to their logarithms. That is, the scale is divided logarithmically, and the designations are not the logarithms but the numbers corresponding to the logarithms. Thus, for instance, 4 is over halfway between 1 and 10 because the logarithm of 4 is 0.6021, which is more than half as large as the logarithm of 10 which is 1, but instead of placing the logarithm 0.6021 on the scale, the corresponding number 4 is put on.

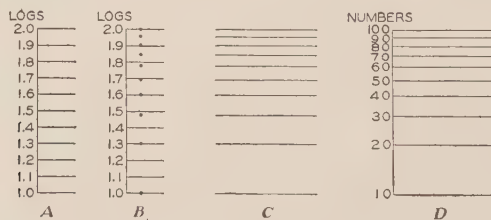
² Semilogarithmic paper, for plotting statistical charts, may be obtained from the Educational Exhibition Co., 26 Custom House Street, Providence, R. I.; Keuffel and Esser Co., 127 Fulton Street, New York City; the Standard Graph Co., 32 Union Square, New York City; John Wenzel, 63 West 107 Street, New York City; Codex Book Co., New York City; and Jesse Ray Miller, 3474 University Avenue, Los Angeles.

³ If semilogarithmic paper or a logarithmic scale is not at hand, a ratio chart may be made by reducing the data to logarithmic terms, and then,

complete cycle or deck on a logarithmic scale is usually numbered from 1 to 10, as shown in Figs. 168 and 169, and the decimal point may be placed so that the scale will be suitable for the curve to be plotted. That is, a cycle or deck may be 0.01 to 0.1, 0.1 to 1, 1 to 10, 10 to 100, or 100 to 1,000, etc. Figures 169 and 170 show examples of two scales. Figure 170 shows a continuous scale through five decks. In all cases, each cycle or deck is ten times as large as the one immediately below. Thus, Fig. 169 shows two complete cycles numbered from 0.1 to 1 and from 1 to 10, or a continuous range of from 0.1 to 10 tons. Figure 170 shows five complete cycles, but only the bottom line of each deck is numbered because of the limited space for lettering. Note, in this figure, that each deck is ten times the one below and that a continuous scale of from 0.1 to 10,000 is given. There are as many lines in a deck in Fig. 170 as there are in Fig. 169, and they represent the same numbers. In Fig. 168, there are more subdivisions ruled in for convenience in plotting, but no more are necessary for reading than are given in Figs. 169 and 170.

using the ordinary arithmetic scale and paper, the logarithms of the given quantities will be plotted instead of the quantities themselves (Fig. 183G).

If, however, no logarithmic scale is at hand, and it is desired to make a chart on a semilogarithmic coordinate ruling, the logarithmic scale may be laid out as follows: Suppose that it is desired to lay out a logarithmic scale in round numbers from 10 to 100 (10, 20, 30, 40, 50, 60, 70, 80, 90, 100). Look up the logarithms of these round numbers, which would be as follows: 10, 1.00; 20, 1.30; 30, 1.48; 40, 1.60; 50, 1.70; 60, 1.78; 70, 1.85; 80, 1.90; 90, 1.95; 100, 2.00. Next, lay out, arithmetically, a scale on which to plot these logarithms as in *A* as illustrated below. Then plot the above logarithms. This is shown by the points in *B*. Next, rule lines spaced according to the points plotted in *B*, as in *C*. Then number the lines according to the



natural numbers as in *D*, not according to the logarithms. As previously explained in footnote 1 on page 133, the divisions of a logarithmic scale are proportional, not to the numbers themselves, but to their logarithms, that is, the scale is divided logarithmically and the designations are not the logarithms, but the natural numbers.

In Fig. 168 it will be noted that the lines are closer together as the scale ascends, and finally some must be omitted.

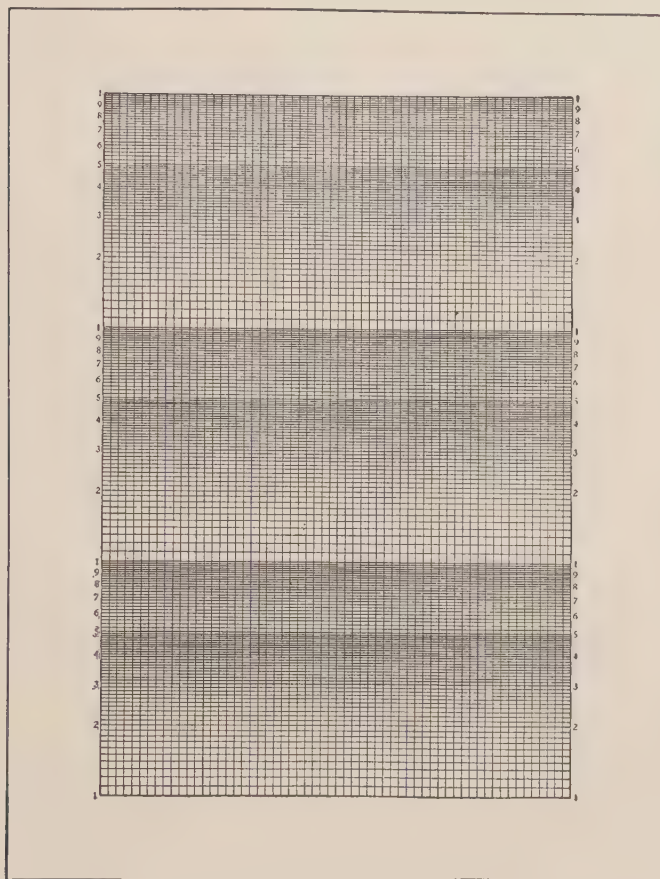


FIG. 168.—Letter size logarithmic paper. (Reduced $2\frac{1}{2}$ times linear.) (Sometimes called semilogarithmic since only the vertical scale is logarithmic.)

Test for the Logarithmic Scale.—A simple test to apply to the logarithmic scale to see whether or not it is laid out correctly is to take the scale, dividers, or an edge of paper, and see if the spaces from numbers to their doubles are the same. That is, the distance from 1 to 2 should be the same as from 3 to 6, or 500 to 1,000, or four millions to eight millions, etc. (Doubles are mentioned for convenience only. One to three, or any other

ratio would do equally well, as equal ratios have equal spaces in all cases.)

The Logarithmic Scale Has No Base or Zero Line.—Since the same space stands for the same ratio of magnitudes in all parts of a given logarithmic scale, obviously we can approach zero indefinitely but can never reach it. There is no zero on a logarithmic scale. Neither is there a base line, for any value of a logarithmic scale is as much base value as any other. The scale

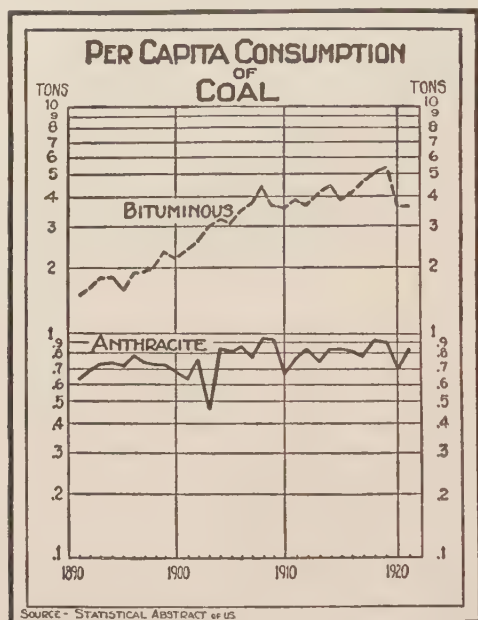


FIG. 169.—Logarithmic chart of two decks. (Courtesy of Federal Reserve Bank of Philadelphia.)

may begin with 1, 2, or any other number. It is necessary only to choose a sufficiently long part of the scale to include the data to be plotted, as in Fig. 171. Since there can be no base, the bottom line of a logarithmic chart should not be emphasized as is the zero line for an arithmetic scale (see Fig. 171).

Significance of Slopes of Curves.—In logarithmic charts the index of the rate of change is the slope of the curve. If a curve has a steeper slope in one part than in another, it means that it is changing at a faster rate. Equal slopes indicate equal rates of change. Thus, in Fig. 172, production of writing paper decreased

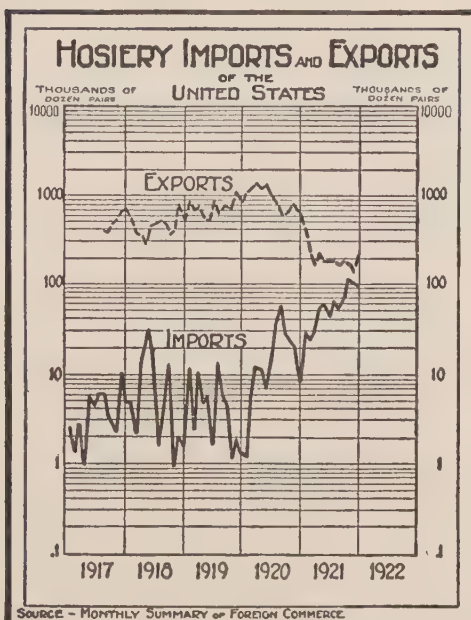


FIG. 170.—Logarithmic chart of five decks. (Courtesy of Federal Reserve Bank of Philadelphia.)

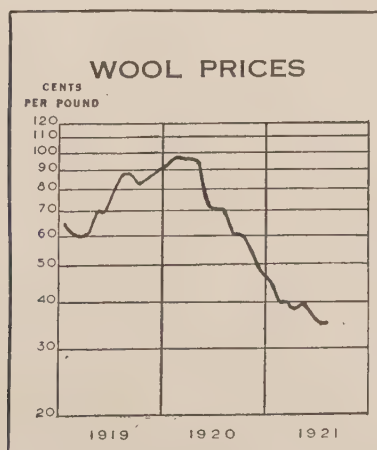


FIG. 171.—There is no base or zero line on the logarithmic scale.

more slowly in the first part of 1919 than it decreased at the end of 1920, and the rate of increase in 1919 is approximately the same as in 1921. A straight line over a period would represent a constant rate of change. The area under the curve, which is so important to an arithmetic curve, means nothing on a logarithmic chart.

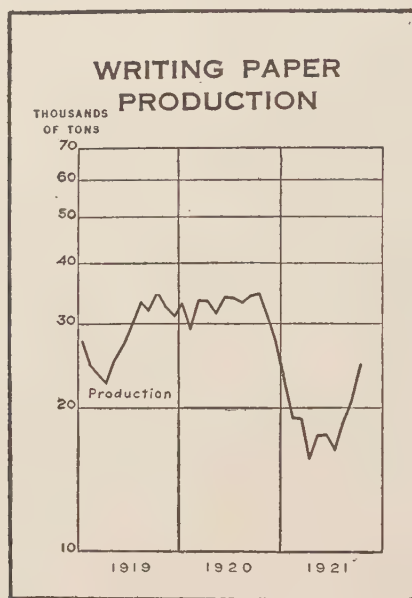


FIG. 172.—The slope of a logarithmic curve indicates the rate of change.

Smoothing Logarithmic Curves.—The rules for smoothing logarithmic curves are the same as those described for arithmetic curves in the preceding chapter.

COMPARISONS OF RATES OF CHANGE IN MORE THAN ONE VARIABLE

It must be kept in mind that the purposes of the arithmetic curve and the logarithmic curve are distinctly different. The two cannot be combined, and one cannot take the place of the other. This will be brought out more clearly by a consideration of parts *A* and *B* of Fig. 173. In *A* we can compare the size of one store with the other, and it is easy to see that the main store is about four times the size of the branch. We cannot, however, directly compare the rates of change in the two curves. That is, we can-

not tell which store is growing the faster. To determine this, we can use the logarithmic curve, as illustrated in part *B* of Fig. 173, which shows that the rates of change are the same. Of course, one can estimate from the scales what the rate of change is in an arithmetic curve, and what the size difference is in a logarithmic curve, but these estimates are made from values on the scales and not from directly reading the curves.

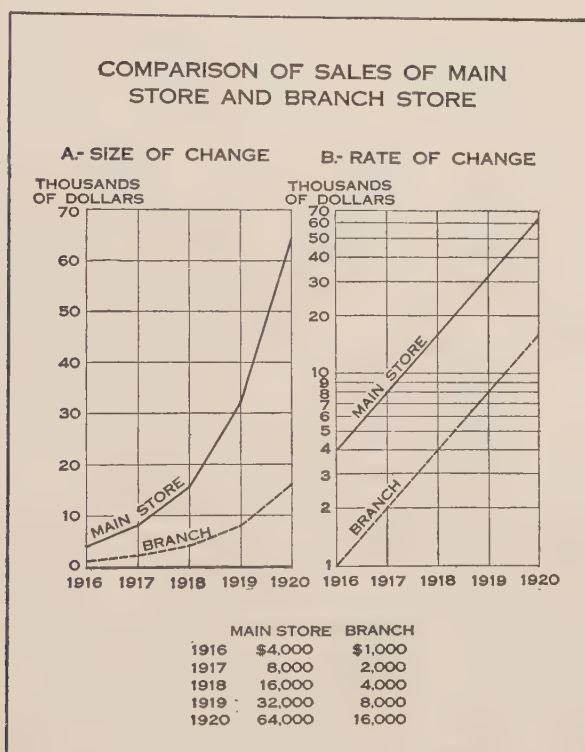


Fig. 173.—Comparison of arithmetic and logarithmic curves.

Business men often attempt to compare the rates of change between arithmetic curves without realizing how greatly they are being misled. For instance, several bankers in a large eastern bank were studying the curves shown in Fig. 174 from a chart which appeared in the *Federal Reserve Bulletin*. After a discussion, they agreed as to the reasons why cotton consumption fluctuates more than wool consumption, when, as a matter of fact, cotton does not fluctuate relatively as much as wool (Fig. 175).

Only Vertical Distances Should Be Compared.—In part *B* of Fig. 173 the rates of change are the same in both series. Whenever, in a logarithmic chart, two curves run parallel in the sense that the vertical distance between them remains unaltered, the rates of change are the same. Note that curves which are

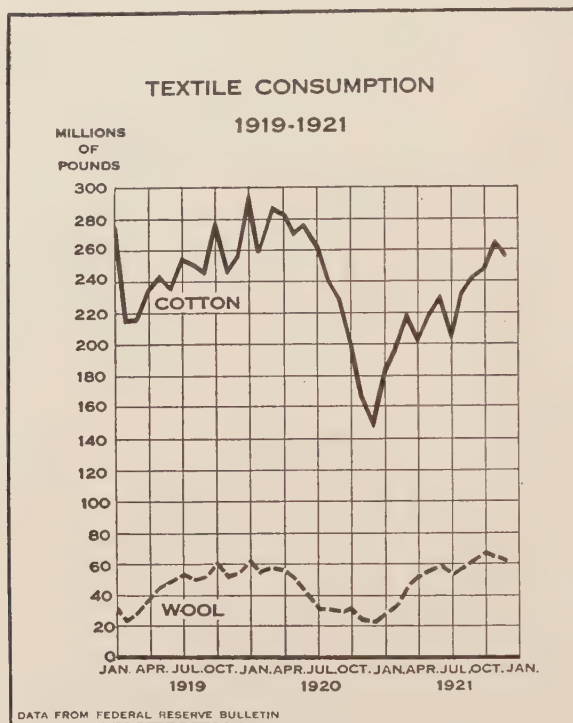


FIG. 174.—Comparing sizes or quantities.

parallel in this sense may not be separated by equal distances (Fig. 176). This effect occurs only when curves are far from straight, but in such cases care must be taken to consider the vertical distances between the curves.

Unless this principle is understood when reading a chart like Fig. 177, which shows the range of prices proportionally, one may be misled by reading the widths of the black areas instead of the vertical distances across the areas. This objection is overcome in Fig. 178 in which the length of the bar represents the proportional ranges. In this case, if print cloth varies from 80 cents to \$1.20, the proportional change is the same as if cotton

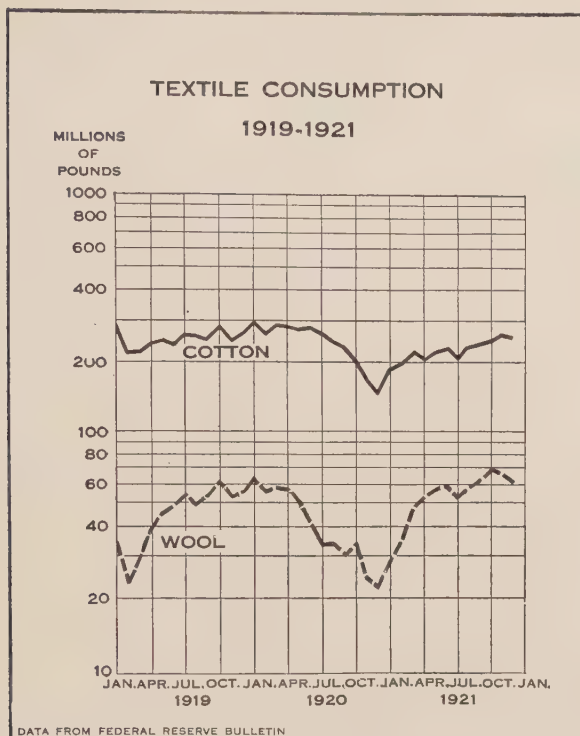


FIG. 175.—Comparing relative fluctuations or rates of change.

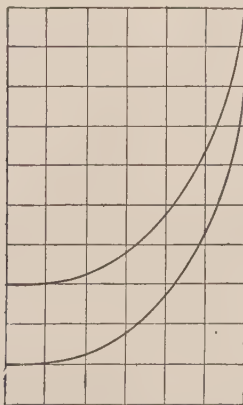


FIG. 176.—These curves are parallel in the sense that the vertical distances between them are the same.

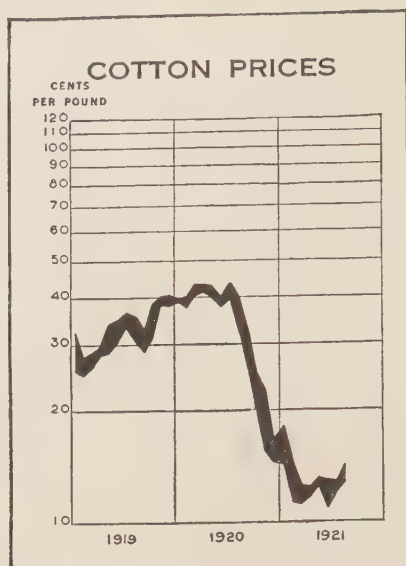


FIG. 177.—Relative range of fluctuations.

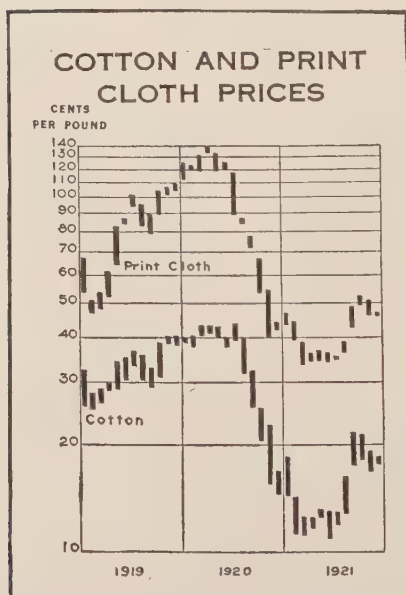


FIG. 178.—Relative range of fluctuations.

varies from, say, 20 to 30 cents. In Fig. 178, the distance from 80 cents to \$1.20 is the same as from 20 to 30 cents, and, therefore, bars representing the same proportional changes would be of equal lengths.

This type of chart (Fig. 178) must not be confused, however, with the bar chart shown in Fig. 179, which is very misleading and should not be used as it does not represent relatives, and it cannot

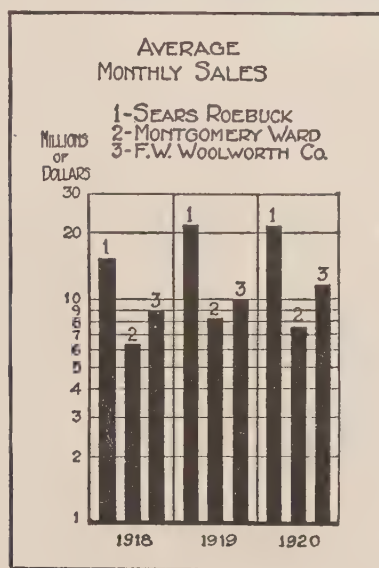


FIG. 179.—Incorrect use of bars on the logarithmic scale.

represent sizes since there can be no zero base in a logarithmic scale. A bar chart comparing size should never be attempted on anything but an arithmetic scale.

Scale Units and Designations.—In making logarithmic charts, it is desirable to include a scale of the units used; this is an aid in checking and a convenience in reading values from the curve. This scale should be expressed in the same kinds of units as are used in an arithmetic chart, and for the same data the two would be alike except that numbers will have to be omitted as the spacing decreases. The rules for designations and for positions of scales are the same for both kinds of charts. The rules are the same also for differentiating and designating the different curves. These points have been discussed at length in the preceding chapter.

Common Units Not Necessary in Comparing Rates of Change.
Rates of change between variables may be compared when they are not expressed in common units. For instance, in an arithmetic chart we cannot compare the *sizes* of coal and oil productions when coal is expressed in tons and oil in barrels. Different units do not prevent comparing *rates of change* on a logarithmic chart, however, for if oil production increases over the

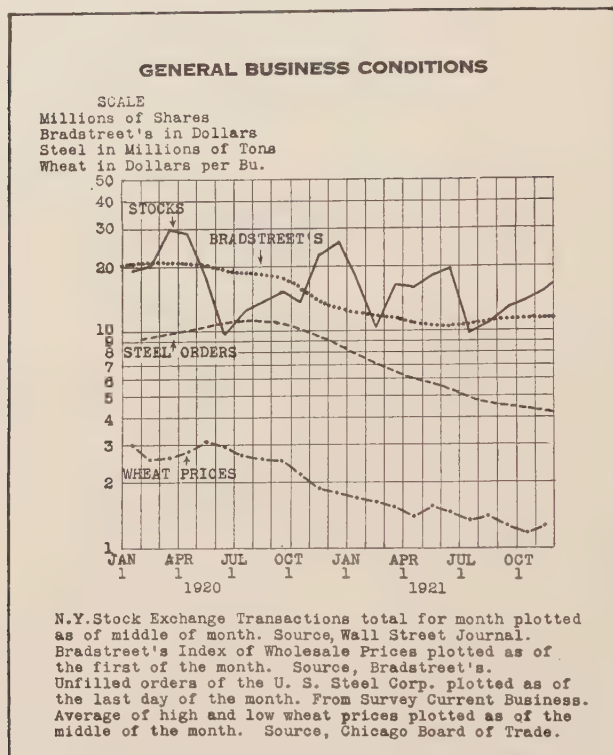


FIG. 180.—Comparison of unlike time distributions.

period from, say, two hundred to four hundred millions of barrels, and coal increases from, say, three hundred to six hundred millions of tons, the production has doubled in both instances, and hence the rates of change are the same. This point is further illustrated in Fig. 180, in which the problem is to compare the four curves with reference to how sensitive they are to business conditions. Different scales are used, which bring the curves close enough together to be compared easily. Not only does this

chart show which curve moves first, but it compares the rates at which it moves as compared with the others, and how far it moves relatively. Thus, for instance, unfilled steel orders and Bradstreet's price index may be compared not only as to which moves first, but as to which moves more rapidly and fluctuates more widely.

Comparing Data of Unlike Time Distribution.—Figure 180 presents also the problem of comparing data of unlike time distribution. Thus, unfilled steel orders are given only for 1 day

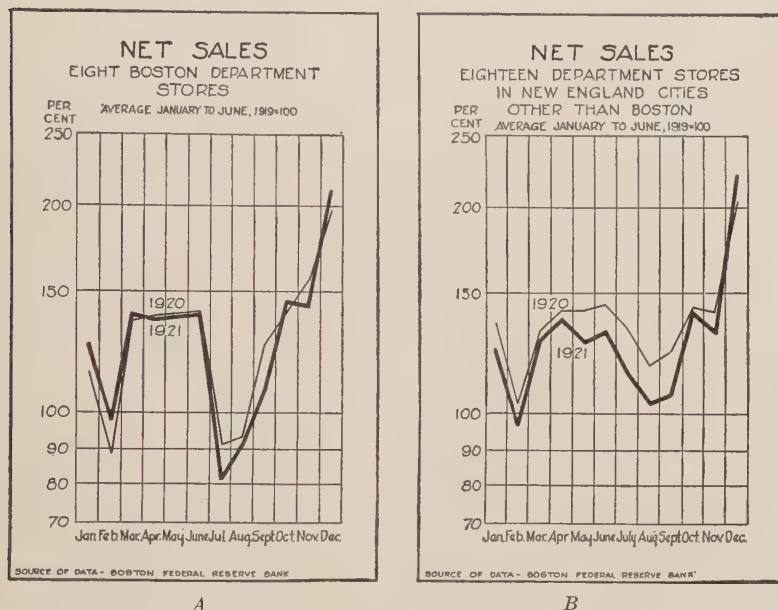


FIG. 181.—Comparison of separate logarithmic charts.

each month (the last day of the month). Bradstreet's price index is given for the first day of the month, but is based primarily on the previous month. New York Stock Exchange transactions are the total number of shares traded during the month, and wheat prices are the average of the high and low for the month. While an exact comparison cannot be made because the data are unlike, often it is desirable to compare the relative movements, and care must be used in plotting, or an apparent lag may not be a true one. In Fig. 180 the monthly totals and monthly averages are plotted as of the middle of the month as the position of least error. Bradstreet's price index is plotted

on the first, and unfilled steel orders are plotted on the first of the following month, that is, for instance, orders for January 31 are plotted on the February 1 line.

Standardization.—When it is desired to make logarithmic charts in such a way that they can be compared directly, they should be drawn to the same scales as illustrated in Fig. 181. Then the rates of change of the curves in chart *A* can be compared with the rates in chart *B* simply by comparing the slopes. It is easy to see that this could not be done if the intervals for

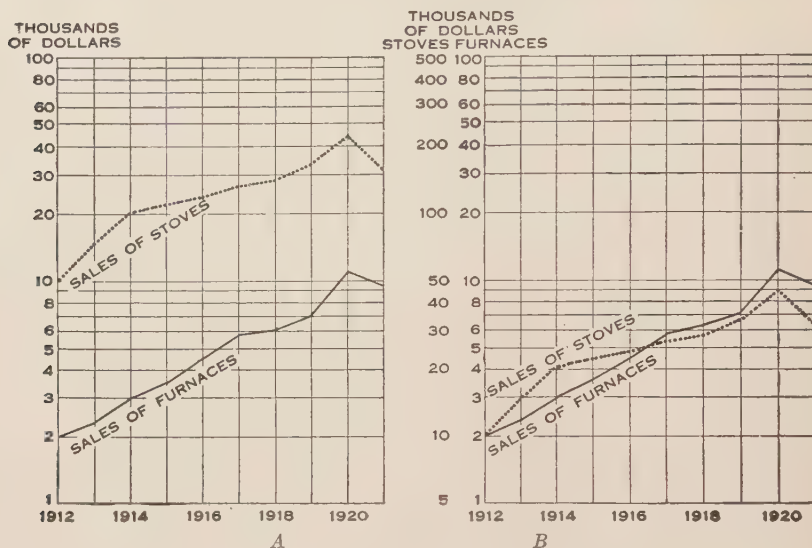


FIG. 182.—Shifting curves to aid comparison.

the same ratios in the *vertical* scale were larger in one chart than in the other, or if the *horizontal* scale intervals in one were larger than in the other.

Shifting Curves for Ease of Comparison.—Since the index of the rate of change is the slope of the curve, the logarithmic chart is sometimes made as in part *B* of Fig. 182 to facilitate ease of comparison. Thus, in part *A* of Fig. 182 the curves are too far apart to see easily the differences in the rates of change, so in part *B* these same curves are placed together and the differences can be noted more easily. Both curves still may be read from the scales if they are included as in part *B* of Fig. 182. If it is desired to make the original plot as in part *B* of Fig. 182, one scale may be raised or lowered in position along the other by

multiplying or dividing it by a constant. Thus, in *B* of Fig. 182, it will be noticed that every number in the left-hand scale is exactly five times the opposite number in the right-hand scale.

Different charts, when drawn to the same scales, may be superimposed upon each other and shifted about until the curves can be compared easily. Such comparison is greatly facilitated by using a glass copying board such as the one illustrated in Fig. 24.

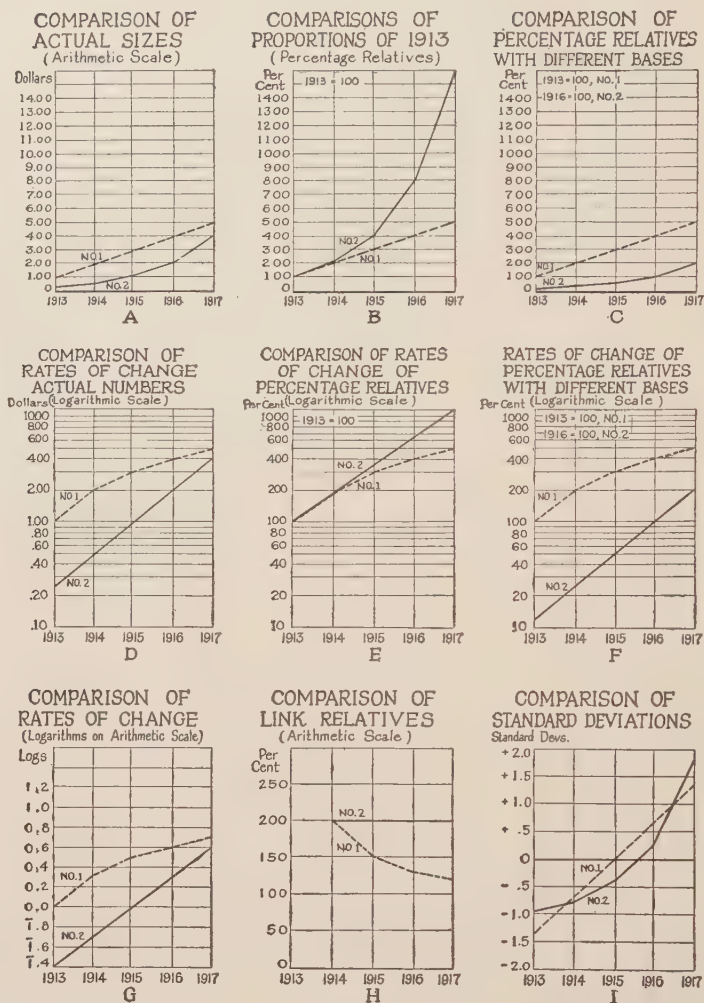
Percentage Relatives.—Percentage relatives are used when it is desired to compare, not absolute sizes of the quantities or rates of change over the period, but the changes that have taken place in terms of percentages of the value at some particular period. In determining percentage relatives, the data for one period or average of a period are taken as the base, usually indicated as 100, upon which the data for the following or preceding periods are computed as percentages.¹ Thus in Fig. 183, *A* compares the actual sizes of the two prices over the period, while *B* compares the changes that have taken place since 1913 by showing what per cent of the 1913 value the prices are for the other dates. That is, for 1915 the percentage relative on a base of 1913 for stock number 2 is 400 (Fig. 183*B*), which means that it is 400 per cent of the same commodity price for 1913.

The percentage relative chart is often used to compare variables that are expressed originally in different units. That is, for instance, we can compare the percentage changes that have taken place in the production of lumber (board feet) and steel (tons) in terms of some particular date.

The use of the percentage relative chart must not be confused with that of the logarithmic chart. The two are distinctly different. While we can see from chart *B* in Fig. 183 that the proportional increase from 1916 to 1917 is greater in number 2 than in number 1, we cannot compare the rates of change for the different periods in either curve. The *rates of change* are compared in Fig. 183 in charts *D*, *E*, *F* and *G*. While curve number 2 has a constant rate of increase, chart *B* would give the impression that the rate is increasing, and while the rate of curve number 1 is falling off, it appears to be constant in chart *B*. On a logarithmic scale, the slopes of the curves are the same

¹ A rapid and easy method for calculating percentage relatives is to use the slide rule with the following setting:

C scale	Base number (24)	Other number (36)
D scale	100	Percentage relative (150)



PRICES OF STOCKS NO. 1 AND NO. 2												
YEAR	DOLLARS		PERCENTAGE RELATIVES Base 1913		PERCENTAGE RELATIVES		LOGARITHMS		LINK RELATIVES		STANDARD DEVIATIONS	
	No. 1	No. 2	No. 1	No. 2	No. 1 Base 1913	No. 2 Base 1916	No. 1	No. 2	No. 1	No. 2	No. 1	No. 2
1913	1.00	0.25	100	100	100	12.5	0.00	1.39			-1.42	-.85
1914	2.00	0.50	200	200	200	25	0.30	1.69	200	200	.71	-.76
1915	3.00	1.00	300	400	300	50	0.47	0.00	150	200	0	-.40
1916	4.00	2.00	400	800	400	100	0.60	0.30	133	200	+.71	+.33
1917	5.00	4.00	500	1600	500	200	0.69	0.60	125	200	+1.42	+1.79

Fig. 183.—Various methods of comparing time series.

whether the actual numbers or the percentage relatives are plotted.

If the percentage relatives of the same base year are plotted as in chart *E* in Fig. 183 the curves are automatically shifted so that they are close to each other, which facilitates the comparison of logarithmic curves as has been discussed previously. It is important to note that this is a convenient method for making this shift. This was the principal reason for reducing the data shown in Fig. 184 to percentage relatives. A slight spread between the curves in Fig. 184 is very significant, and unless they

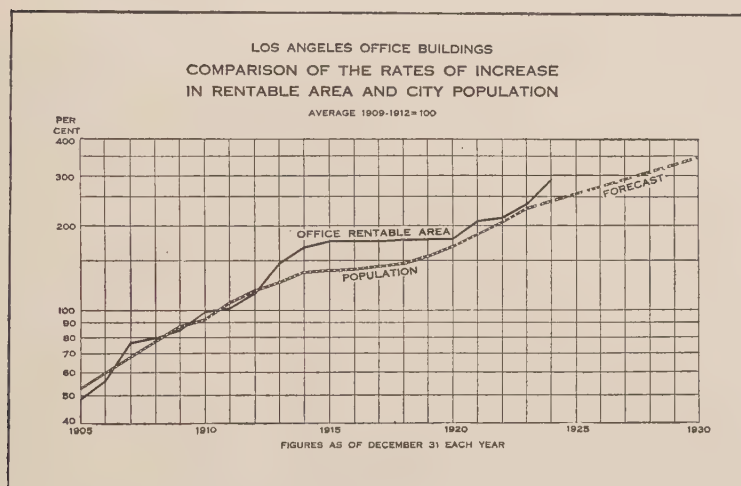


FIG. 184.—Use of percentage relatives for bringing curves together to facilitate comparison.

are close together it is very difficult for the reader to appreciate the differences in their slopes.

If two series of percentage relatives, having different base periods, are to be studied, they cannot be compared directly on an arithmetic scale without distortion (see chart *C*, Fig. 183). Different base periods, however, make no difference in the shapes of the curves plotted on the logarithmic scale; this is illustrated by chart *F* in Fig. 183. These different methods of handling the data do not alter the rates of change, and hence there is no difference in what is shown by charts *D*, *E*, and *F* of Fig. 183. Thus, if we wish to compare the United States Bureau of Labor Index

of Wholesale Prices with Sauerbeck's Wholesale Price Index, which has a different period as a base, we may compare the rates of change by plotting them directly on a logarithmic chart.

If a logarithmic scale is not at hand, a logarithmic chart may be constructed as shown in chart *G*, Fig. 183. In this chart the data are reduced to logarithmic terms and then, using the ordinary arithmetic scale, the logarithms of the given quantities are plotted instead of the quantities themselves. Rates of change are shown by the slopes of the resulting curves. It will be noted that the shapes and slopes of the curves in chart *G* of Fig. 183 are the same as in charts *D*, *E*, and *F*.

The Simple Percentage Method.—It is often attempted to show rates of change by another percentage method, and when curves do not greatly diverge, this method gives approximately correct results, but if the curves diverge greatly, comparisons of

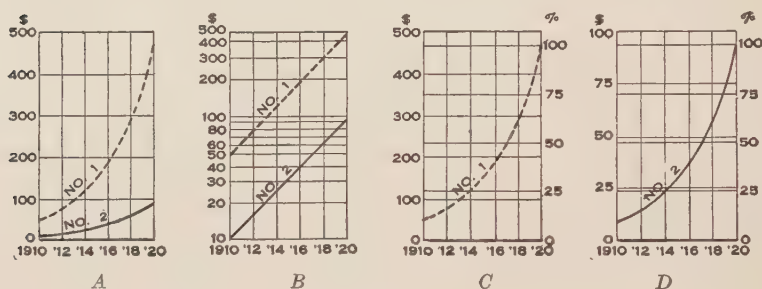


FIG. 185.

rates of change are likely to be very misleading. Parts *C* and *D* of Fig. 185 illustrate this method, which has been used widely in an attempt to combine the advantages of both the arithmetic and the logarithmic chart. For the sake of simplicity in illustration, the data shown under "First Assumption" in the table on page 151 and plotted in Fig. 185 represent two investments made in 1910, one of \$50 and the other of \$10 and the rate of increase is the same for both, namely 25 per cent per annum compound interest. Figure 185*A* shows the data plotted on an arithmetic scale, while *B* compares the rates of increase. Rates of change can be compared roughly on the arithmetic scale between parts of curves that are approximately the same distances from zero, and based upon this principle 100 per cent is given the same space from zero for each curve. Thus (in Fig. 185*C*) \$465.66 equals 100

FIRST ASSUMPTION

Year	Investment Number 1	Investment Number 2
1910	\$ 50.00	\$10.00
1911	62.50	12.50
1912	78.13	15.63
1913	97.66	19.53
1914	122.07	24.41
1915	152.59	30.52
1916	190.74	38.15
1917	238.42	47.69
1918	298.02	59.61
1919	372.53	74.51
1920	465.66	93.13

SECOND ASSUMPTION

Same as above except

	Investment Number 1	Investment Number 2
1920	same	\$465.66

per cent while in *D* \$93.13 equals 100 per cent, and the intermediate values are located proportionally to these totals. Since both a scale of dollars and a percentage scale are given in *C* and *D*, both values (left-hand scale) and percentages (right-hand scale) can be read from the same chart.

It is the purpose of the users of charts like *C* and *D* (Fig. 185) to combine the advantages of *A* and *B*, but as a matter of fact the difference in size, which is the purpose of the arithmetic chart *A*, cannot be compared graphically in *C* and *D*. In regard to the rates of change, we see that in *C* and *D* the curves are alike, but, while we can compare any period in *C* with the same period in *D*, we cannot compare the rates of change between any two periods of the same curve in *C* and *D* as we can in *B*.

The rates of change are shown to be equal for the same periods in *C* and *D* (Fig. 185), since the curves have the same slopes, but they will be alike only in such an ideal case. Let us assume

that the value of Investment Number 2 increases according to the "Second Assumption" in the preceding table from \$74.51 to \$465.66 in 1920, instead of to \$93.13. Now the arithmetic curves will appear as in Fig. 186*A* and the logarithmic curves will appear as in Fig. 186*B*. The rates of change are the same in both series until 1919. Using the percentage method above described, the curves would appear as in Fig. 186*C* and *D*. This percentage method now fails as far as comparing rates of change is concerned. While the rates of change are the same up until the last year, they are shown by the percentage charts *C* and *D* (Fig. 186) no better than by chart *A*, which is not intended to show rates of change at all. The figures charted in Fig. 186 represent an extreme assumption, but no more extreme than the

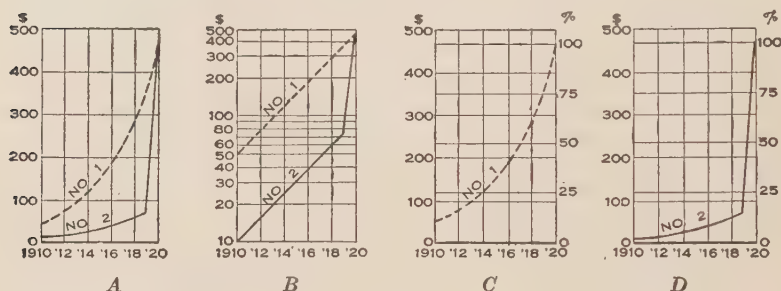


FIG. 186.

ideal case shown in Fig. 185. Figure 186 brings out the dangers of juggling the percentage scale in an attempt to show rates of change.

Standard Deviations.—In a problem such as that of showing business cycles graphically, when studying the relationship between different series, a situation arises which cannot be solved directly by any of the graphic methods previously discussed. Suppose, for example, that we wish to compare two series, and that in one the percentage deviations range from +17 to -22 per cent, while in the other the range is from +28 to -44 per cent, as shown in chart *A* of Fig. 187. In such a comparison, a fluctuation of 10 per cent in bank clearings has an entirely different significance than has a fluctuation of 10 per cent in pig iron production. In certain studies of business cycles, it is undesirable to have these differences in amplitude. In such cases, each series should be reduced to standard deviations, which method

CHART A: COMPARISON WITH NORMAL - PIG IRON PRODUCTION AND BANK CLEARINGS OUTSIDE NEW YORK CITY

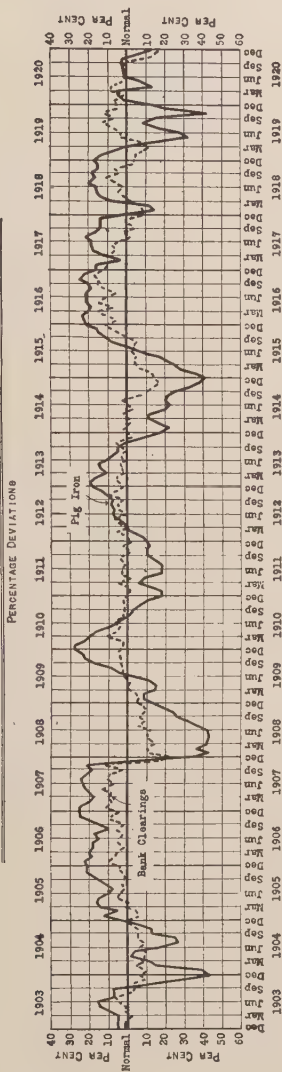


CHART B: COMPARISON WITH NORMAL - PIG IRON PRODUCTION AND BANK CLEARINGS OUTSIDE NEW YORK CITY

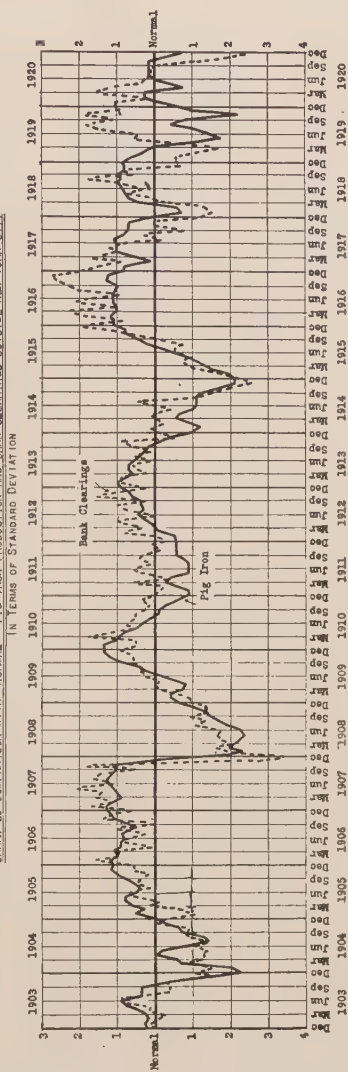


Fig. 187.—Use of standard deviations in graphic presentation. (Courtesy of American Telephone and Telegraph Co.)

is the best measure of dispersion for most purposes in common use at present. Chart *B* of Fig. 187 shows the data of chart *A* plotted in terms of their standard deviation.¹

¹ For the computation of standard deviations, see any standard text on statistics such as YULE, G. U., "Introduction to Statistics," Chap. VIII; KING, W. I., "Elements of Statistical Methods," Chaps. XIII and XIV; SECRIST, HORACE, "Introduction to Statistical Methods," Chap. XI; DAVIES, GEORGE R., "Introduction to Economic Statistics," Chap. II; RIEGEL, ROBERT, "Elements of Business Statistics," Chap. XII; MILLS, FREDERICK C., "Statistical Methods," Chap. V; JEROME, HARRY, "Statistical Method," Chap. IX; or KENT, FREDERICK C., "Elements of Statistics," Chap. VIII; CHADDUCK, ROBERT E., "Principles and Methods of Statistics," Chap. IX; SUTCLIFFE, WILLIAM G., "Elementary Statistical Methods," Chap. XV; DAY, EDMUND E., "Statistical Analysis," Chap. XI.

CHAPTER VII

FREQUENCY DISTRIBUTIONS

It is the purpose of this chapter to describe the graphic methods of showing "frequency distributions." Both bars and curves are used to represent frequency distributions, and the forms are similar to the time curves described in the preceding chapters.

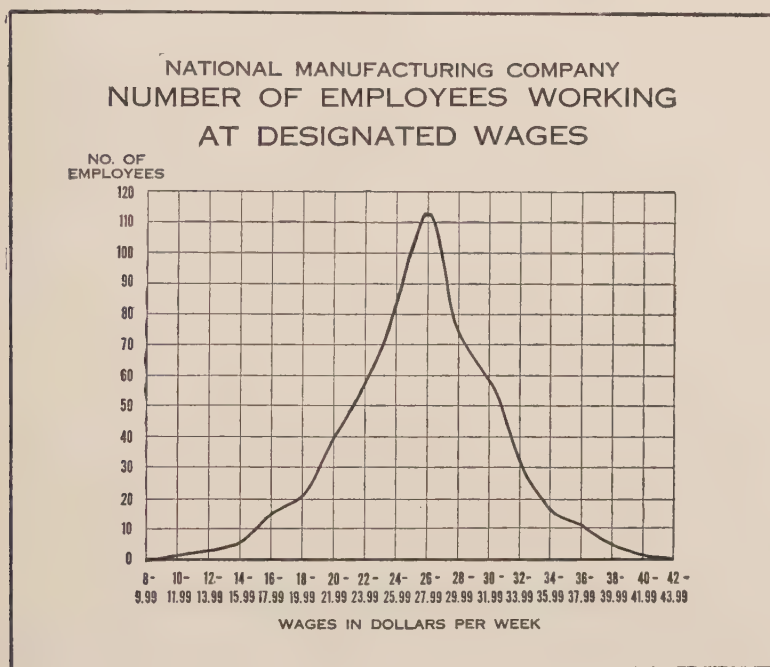


FIG. 188.—A frequency curve.

Thus, in Fig. 188 the vertical scale represents the number of times that each measurement occurs, and the horizontal scale represents the different measurements arranged in order. When bar charts are used to show frequency distributions, it is desirable to use vertical bars, rather than horizontal, so that they may be read in the same direction as curves (Fig. 194).

Scales for Frequency Graphs.—Equal distances on the scale of a frequency graph should represent equal values. Both scales should be divided into units of round numbers so that they may be read easily. The positions of scales and their designations are illustrated in Fig. 188. That is, they should be outside of the coordinate lines—below and to the left. If desirable to have scales above the graph and on the right, they may be placed there in addition to the scales underneath and on the left, but in no case should the scales underneath and on the left be omitted.

The vertical scale of a frequency graph should begin with zero, but the use of zero on the horizontal scale is not necessary unless the range of classes includes the interval whose lower limit is zero.

In these respects the frequency curve is much like the arithmetic time curves previously described. The vertical distances from the zero base to the curve represent the number of occurrences, and unless the whole scale including zero is used, a wrong impression will be given, as explained in Chap. V.¹ The horizontal scale shows the class intervals; and usually the inclusion of zero in this scale, when it is a long way from zero to the first class interval for which there are values, adds nothing and may detract from the value of the graph.

Class Intervals.—In general, the class intervals of a frequency graph should be equal. Otherwise the chart is likely to give a wrong idea of the distribution (compare Figs. 189 and 190). However, when plotting cumulative curves, this general rule does not prevent the use of smaller intervals for parts of the range over which the frequency varies rapidly, but the scale must be spaced accordingly (Fig. 202 and page 169). In all instances equal spaces should represent equal values. Quite often official statistical publications use class intervals of unequal magnitudes, and it is difficult if not impossible to present the data in the form of a frequency graph. There are, however, occasional exceptions to the rule that class intervals should be equal. It sometimes happens in actual work that class intervals of uniform size are not practical. For instance, in a marketing survey, which made necessary the classification of consumers by income groups, one class interval was \$1,750 to \$2,000 and another was \$4,000 to \$5,000. The difficulty here is with the unit. A dollar means more in the first group than in the second. Persons in the first

¹ See pages 87 to 89 inclusive.

group will try as hard to save \$1 as those in the second will try to save \$4. In relation to the particular commodity under consideration, variations in standards, customs, etc., in the second group were no greater than in the first and, therefore, the purchasing group placed in the \$4,000 to \$5,000 class was as uniform as the \$1,750 to \$2,000 group.

The number of class intervals to be shown on a frequency graph depends upon the character of the data. Usually it is best to

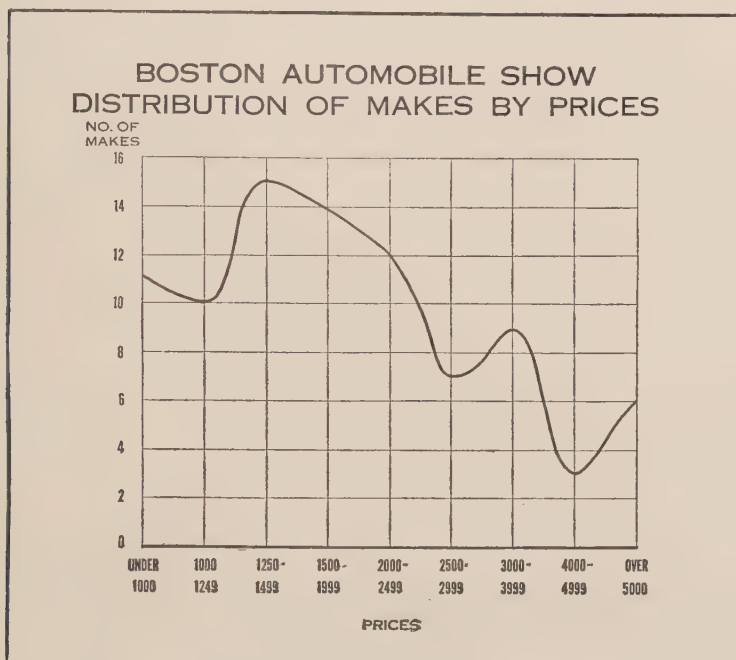


FIG. 189.—Incorrect because of unequal class intervals.

so choose the interval that the entire number of classes is between fifteen and twenty-five.¹ A number of classes less than, say, ten, leads in general to very appreciable inaccuracy, and a number of over, say, thirty, makes an unwieldy graph. This, of course, is not an inflexible rule. The classes should be wide enough to make the distribution fairly smooth, that is, vacant classes should not occur except at the extremes of the range, yet the classes should not be so wide as to obscure the tendencies of the distribution. A class interval should not be omitted from a

¹ YULE, G. U., "An Introduction to the Theory of Statistics," p. 79.

series simply because there are no occurrences in that class. All of the intervals should be included, and the curve or bar brought down to zero when no items occur (see Fig. 190).

Designating Class Intervals.—The most common method of designating class intervals in the horizontal scale is by expressing the lower and upper limits of each class as illustrated in Fig. 190. The limits expressed in Fig. 190 are, of course, inclusive. That is, for instance, the number of prices that falls between 500

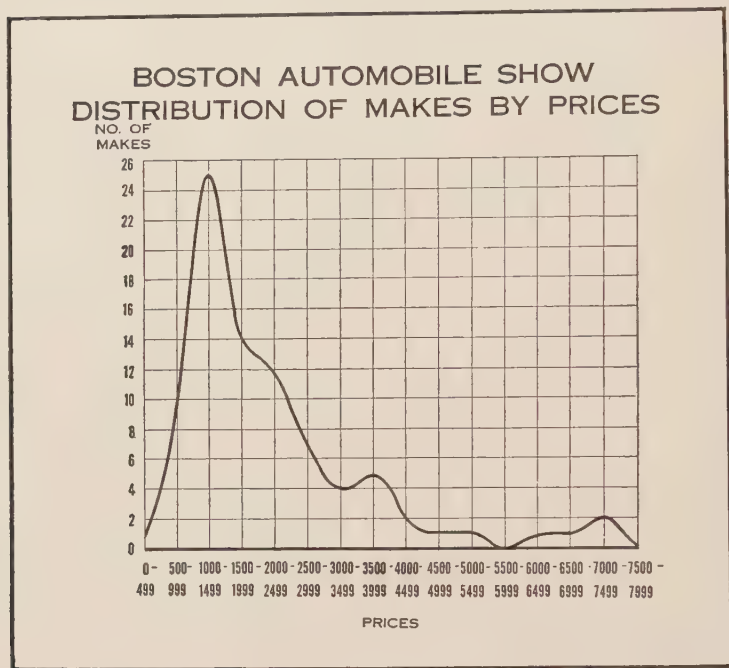


FIG. 190.—Correction of Fig. 189. Class intervals equal.

and 999 inclusive will be plotted above this class. Note that the method illustrated is better than the method of expressing the classes as 500 to 1,000, 1,000 to 1,500, 1,500 to 2,000, etc., because in this latter plan it is not understood generally in which group 1,500, for instance, would fall—the 1,000 to 1,500 interval or the 1,500 to 2,000. Another method of designating class intervals is illustrated in Fig. 191. Here any two adjoining figures designate the limits of a class, and the frequency point is plotted above the center of the class so designated. This method, how-

ever, is not as clear to the average reader as the method illustrated in Fig. 190.

Occasionally, in plotting business data, the range of variation is so small that there will be no class intervals but the unit will serve instead. Or if there is a unit width of class, it is sometimes designated by a single figure as, for instance, \$4, which would include all of the items from \$3.50 to \$4.49 inclusive, or 10 years, which would include all from 9.5 to 10.49 years inclusive, as illustrated in the accompanying chart (Fig. 192).

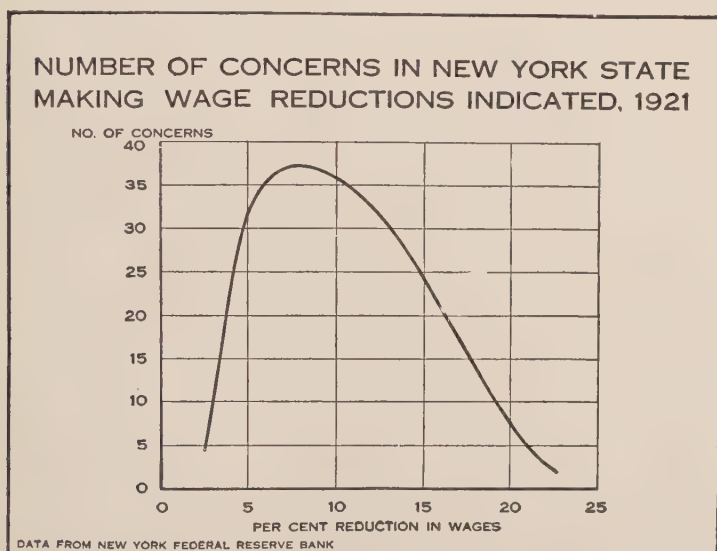


FIG. 191.—Designating class limits by adjoining figures. (Not common practice.)

Continuous and Broken Series.—In making a frequency graph, the form and treatment depend upon whether the series is continuous or broken. A continuous series is one in which measurements occur at all possible points. A broken series is one in which the frequencies occur only at certain points which are determined by the nature of the units. Thus, weekly wages of workmen, if expressed in units no smaller than 25 cents, would be a broken series, while their ages would be a continuous series as they would occur at any possible point within the range.

In the field of business the broken series is common. Prices, wages, and rates usually are fixed in convenient units. Sales

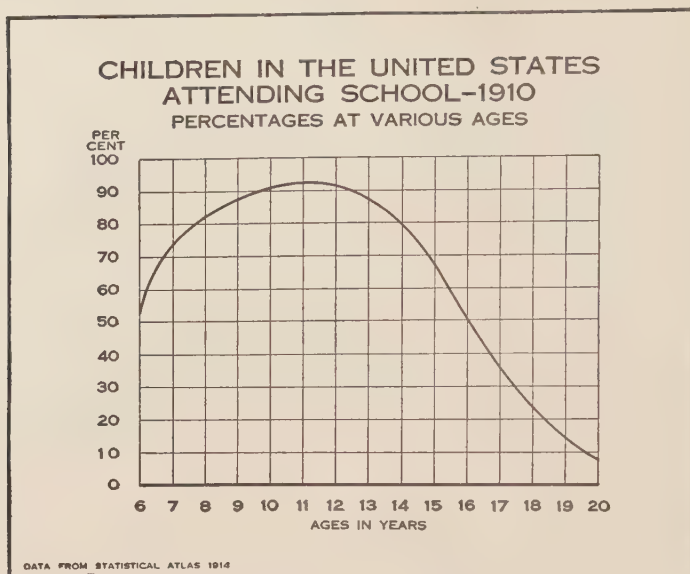


Fig. 192.—Designating class limits in unit widths.

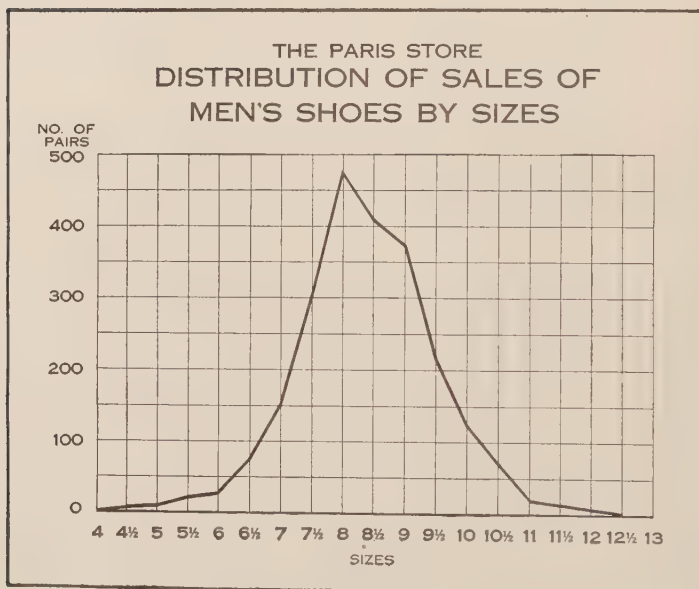


Fig. 193.—Broken frequency curve.

and purchases tend toward round numbers, and not fractions of pieces or fractional weights. Often in this field, series that really are of the continuous type are recorded as broken series. Thus, age will be recorded to the nearest year, weight to the nearest ton, values to the nearest dollar, etc.

If the above two series are to be represented accurately, two different kinds of curves are required, namely, smoothed and broken.

Smoothed and Broken Curves in Frequency Graphs.—When plotting a broken series, the frequency of occurrence of each class

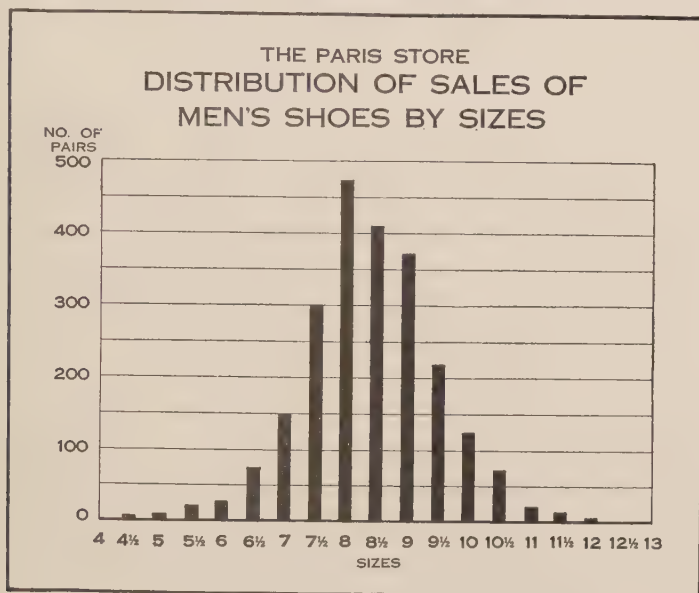


FIG. 194.—A frequency bar chart.

is represented by a point plotted on the vertical line that passes through each class. These points are connected by straight lines, making a broken curve as shown in Fig. 193.

When plotting a broken series, the lines connecting the points have no other use than to aid the eye in comparing their respective heights. There are no values represented on the lines between the points for there can be no occurrences there. The smoothed curve should be used only when it will be instrumental in giving a more accurate idea of the distribution, and thus it should not be used when plotting a broken series, since it implies a continuous

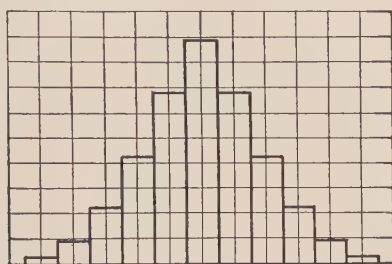
distribution and that the larger the number of observations the fewer will be the number of irregularities. The distribution of a broken series does not change gradually but there is an abrupt change from one class to another, and the direction of this change is shown by the lines from point to point in a broken curve.

While the broken curve is the most common method of showing broken series, the bar chart (Fig. 194), probably is more easily understood by untrained readers. When the significance of the broken curve is likely to be misunderstood, this simple bar chart, in which the frequencies of the different classes and their distribution are indicated simply by the lengths of the bars, will be more satisfactory.

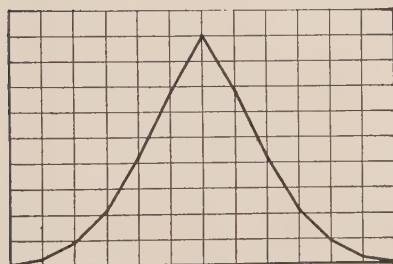
Although the use of the smoothed curve for a broken series is deceptive, it is much more significant of the distribution of a continuous series than either the broken curve or the bar chart. In a continuous series, since variations from one extreme measurement to the other are regular and gradual, the curve should be free from sharp angles, the shape being influenced at each point by the positions of the adjoining points (Figs. 191 and 192).

Smoothing a Frequency Curve.—When smoothing a curve, care should be taken to keep the area under the curve the same as if a column diagram (Fig. 195A), or a broken curve or frequency polygon (Fig. 195B), were constructed. In the diagram of Fig. 195A, the total area represents the total number of observations, or a certain area will represent a certain number of observations. For instance, if two spaces on the horizontal scale represent \$10, and two spaces on the vertical represent 100 occurrences, $100 \times 10 = 1,000$, or the number of observations represented by an area of four spaces. The total area under the curve in Fig. 195B is the same as that in Fig. 195A. This is illustrated by Fig. 195C, which shows that the areas cut off of a column when a broken curve is drawn are the same as the areas added by the curve to the adjoining columns. While the total area is correct in both A and B of Fig. 195, the area over any one class is not correct for a continuous series in the frequency polygon of Fig. 195B. For instance, the middle class has lost without gaining anything (Fig. 195C). This must be considered when smoothing the curve, and usually the best method is to draw the curve so that the areas gained and lost by each column will be equal (Fig. 196). Thus the top of the curve will slightly overtop the highest plotted point. A little thought at this point will indicate that

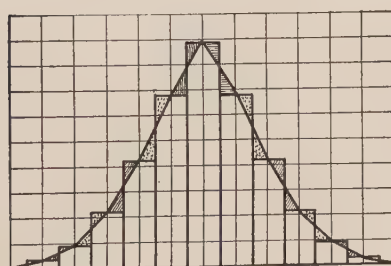
it is reasonable to expect such a distribution within this particular class interval. In actual practice, the column diagram is not usually made as a basis for drawing the smoothed curve, but the curve is sketched directly from the plotted points, allowances



A



B



C

FIG. 195.—The relation between a column diagram and a broken curve.

being made by keeping in mind the area distribution as above described. If the series begins and ends with an infinitely small number of occurrences, the curve should begin and end on the zero line as illustrated in Fig. 196.

It must be remembered that business data are full of irregularities, and that smoothing must not obscure any except minor irregularities which are not thought to actually exist.¹ If an irregularity is such that it would not be eliminated by further sampling, it should not be smoothed out. If smoothing is freely

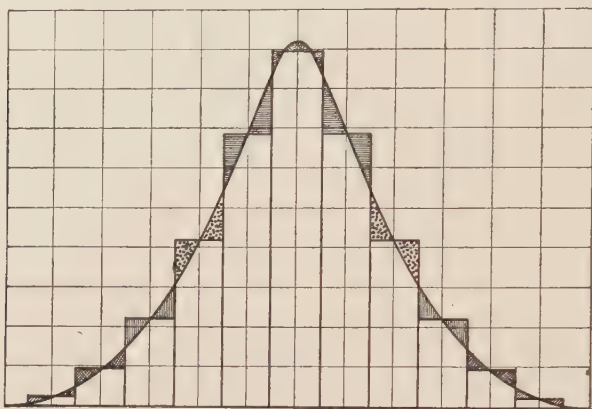
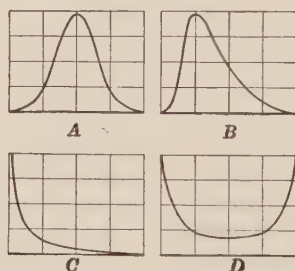


FIG. 196.—Smoothing a frequency curve.

indulged in, it is well to indicate the points from which the curve was made (Fig. 197). Absolute rules, however, cannot be laid down for smoothing frequency curves, for each problem must be considered according to the representative character of the data, and the curve should be plotted which will best suit the purpose for which the graph is made.

¹ While the "bell-shaped curve" is known as the normal frequency curve (A), business data more commonly describe some other form or combination of forms. Yule considers the fundamental simple types of distribu-



tions as four in number,—namely, the symmetrical (A), the moderately asymmetrical (B), the J-shaped (C), and the rarely occurring U-shaped (D). (YULE, G. U., "Introduction to the Theory of Statistics," pp. 87–105.)

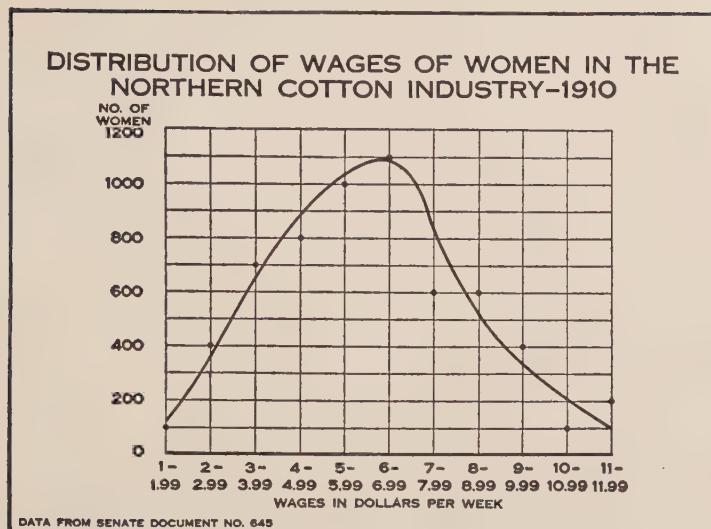


FIG. 197.—Showing points when curve is freely smoothed.

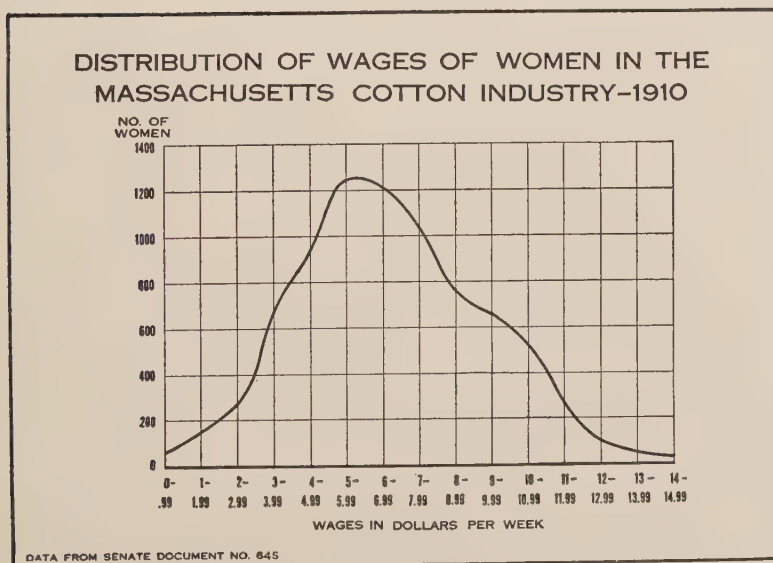


FIG. 198.—Simple frequency curve of data shown in the table on page 166.

Cumulative Frequency Curves.—So far, only simple frequency series have been considered, such as Fig. 198 which is plotted from the first column of the table below. Another form which is used a great deal in presenting business data is the cumulative frequency curve.¹ Frequency series are made cumulative when successive frequencies are added together so that each class includes all the lower or upper classes, depending upon from which end of the range the cumulating process is begun.

DISTRIBUTION OF FEMALE EMPLOYEES IN THE COTTON INDUSTRY
IN MASSACHUSETTS ACCORDING TO WAGES PAID, 1910.
(8,003 OBSERVATIONS)

(From *Senate Document* 645, "Cotton and Textile Industry," page 665)

Wages in dollars per week	Simple frequency	Number of employees	
		Cumulative "Less than" ²	Frequency "More than" ²
0- 0.99 ¹	69	69	8,003
1- 1.99 ¹	150	219	7,934
2- 2.99	264	483	7,784
3- 3.99	688	1,171	7,520
4- 4.99	936	2,107	6,832
5- 5.99	1,242	3,349	5,896
6- 6.99	1,211	4,560	4,654
7- 7.99	1,035	5,595	3,443
8- 8.99	739	6,334	2,408
9- 9.99	689	7,023	1,669
10-10.99	528	7,551	980
11-11.99	245	7,796	452
12-12.99	96	7,892	207
13-13.99	72	7,964	111
14-14.99	39	8,003	39

¹ Number of employees adjusted for first and second classes.

² "Less than" applies to and includes the upper limit of the class, while "more than" applies to and includes the lower limit.

¹ See BRINTON, W. C., "Graphic Methods for Presenting Facts," pp. 149-199, for further examples of the use of this curve.

It makes no difference from which end the cumulating is begun, but if it proceeds from the lesser to the greater, the corresponding frequencies are read "less than," as in Fig. 199, and, if it proceeds from the greater to the lesser, the frequencies are read "more than," as in Fig. 200. In a table like the one on page 166, when cumulations are read "less than" they refer to the upper limits of the classes, and when they are read "more than" they refer to the lower limits. It should be noted also that the limits are included when intervals such as those in Figs. 199 to 202 are used.

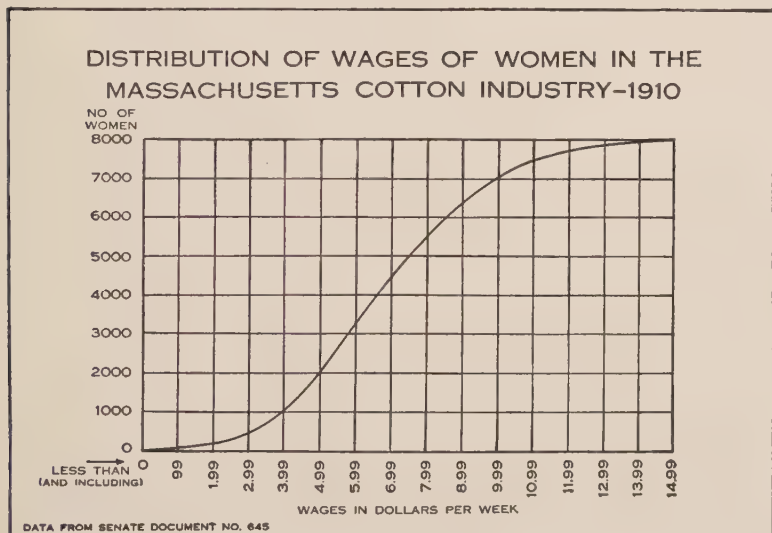


FIG. 199.—A "Less Than" cumulative frequency curve. (See table on page 166.)

Note also that a curve of the "less than" form extends from the lower left to the upper right, and that a "more than" curve extends from the upper left to the lower right as illustrated in Figs. 199 and 200. In either case, as the curve approaches the vertical, relatively higher frequencies are represented.

A graph may be made to read both ways by including two scales on the diagram, as illustrated in Fig. 201, in which one scale is read from the bottom up and the other is read from the top down.

Both continuous and broken series may be plotted cumulatively, but the fundamental differences, which were discussed in connection with the simple frequency curves, must be kept in

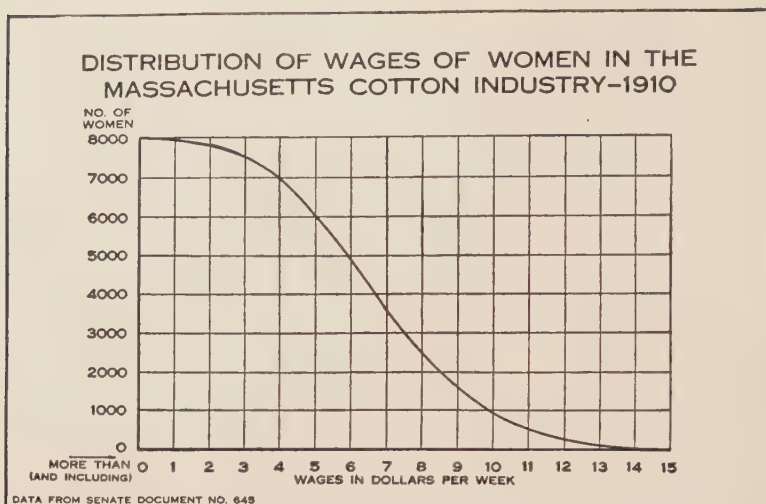


FIG. 200.—A “More Than” cumulative frequency curve. (See table on page 166)

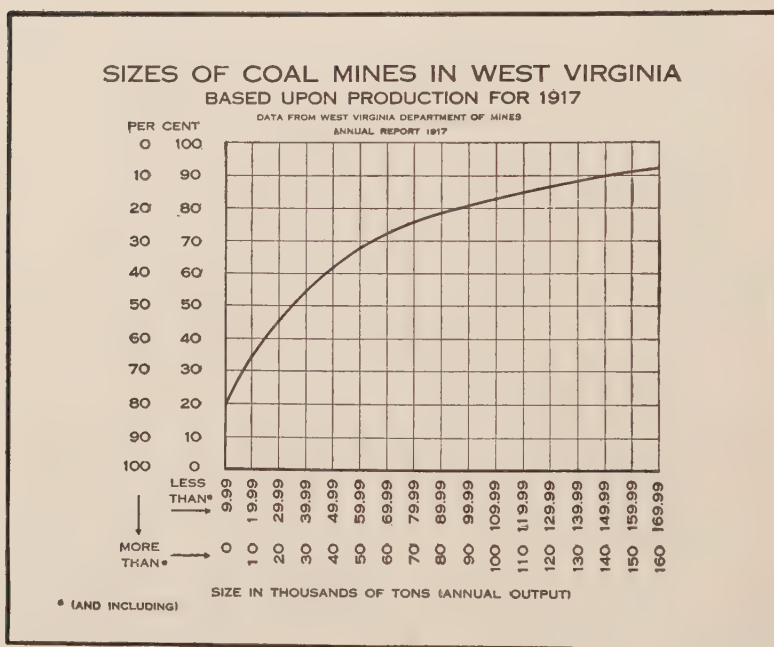


FIG. 201.—A combination “Less Than” and “More Than” cumulative frequency curve.

mind. The discussion on the use of smoothed and broken curves for showing simple frequency distributions applies equally well to cumulative frequencies.

Figure 202 shows a cumulative curve plotted from data which were not given in classes of equal widths because over part of the range the frequencies varied so rapidly. Though not all intervals are of the same size, the curve is so plotted that equal spaces represent equal values and, therefore, the curve is not distorted or misleading.

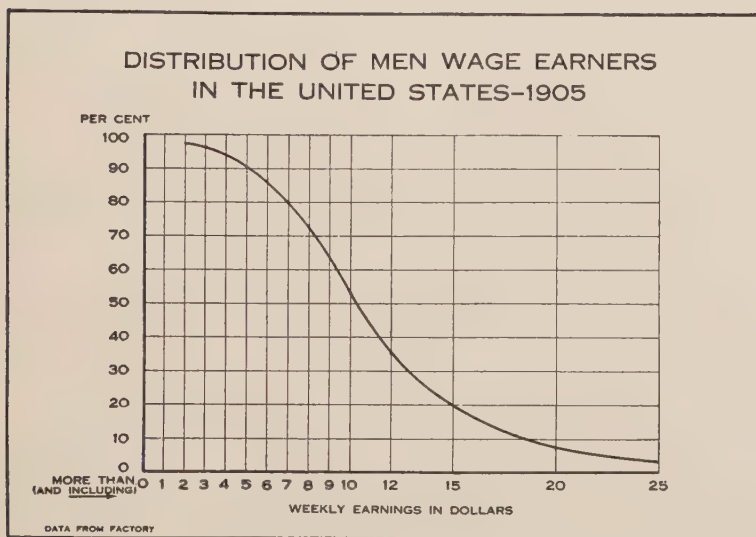


FIG. 202.—A cumulative frequency curve with unequal class intervals.

Comparisons of Frequency Distributions.—If several frequency distributions are to be compared, they are usually plotted on the same chart, and it is best to use either a broken or smoothed curve, depending upon the nature of the series, rather than a column diagram or bar chart.

A comparison of two distributions is illustrated in Fig. 203, which shows the curves plotted according to the actual numbers of occurrences. It will be noticed that the differences in the altitudes of the curves interfere with the comparisons of the distributions, and often it is advisable to place the large and small groups on an equal basis by reducing the frequencies to percentages as in the table on page 171, and plot as shown in Fig.

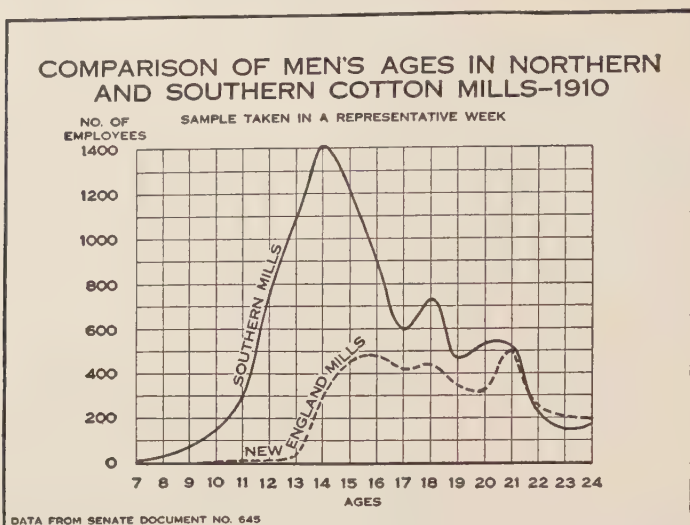


FIG. 203.—Comparison of frequency distributions.

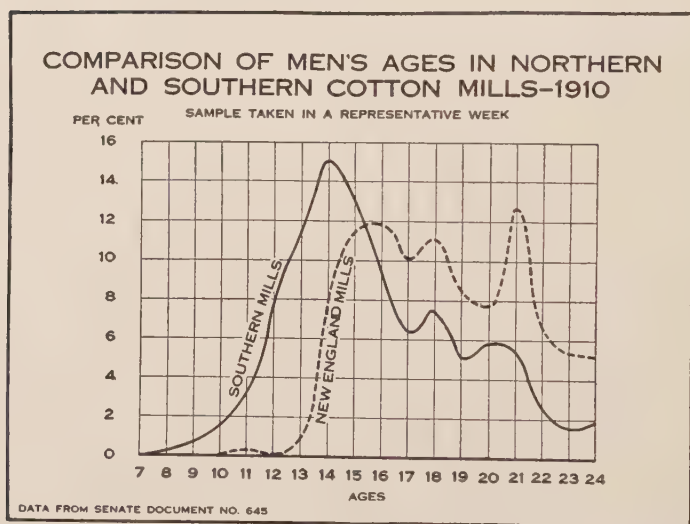


FIG. 204.—Comparison of frequency curves on a percentage basis.

204, in which the total number of wage earners in each section represents 100 per cent and the percentage that each number is of the total is plotted.

DISTRIBUTION OF MALE EMPLOYEES UNDER 25 YEARS OF AGE IN
UNITED STATES COTTON MILLS

(Sample taken from *Senate Document 645*, "Cotton and Textile Industry,"
page 617)

Age in years	New England States		Southern States	
	Number	Per cent	Number	Per cent
7	0	0	7	0.07
8	0	0	29	0.31
9	0	0	65	0.69
10	1	0.03	173	1.84
11	13	0.32	286	3.04
12	5	0.13	760	8.09
13	41	1.03	1,080	11.50
14	304	7.62	1,427	15.19
15	457	11.45	1,227	13.06
16	470	11.77	897	9.55
17	410	10.27	592	6.30
18	444	11.12	736	7.84
19	341	8.54	471	5.02
20	311	7.79	538	5.73
21	507	12.70	519	5.53
22	261	6.54	248	2.64
23	215	5.38	159	1.69
24	212	5.31	179	1.91
Total	3,992	100.00	9,393	100.00

When cumulative curves are being compared, better results can be obtained in many instances if the frequencies are reduced to percentages, since proportional size changes of the frequencies and the regularity of the distribution can be appreciated more readily.

CHAPTER VIII

CORRELATION

“Correlation” is a term employed by statisticians to denote causal relations between series, classes, or groups of data. In presenting correlation graphically, it is necessary to distinguish between simple or “primary” correlation which may exist between two variables, and the more remote or “secondary” correlation which may exist between two variables which are correlated to a third factor.

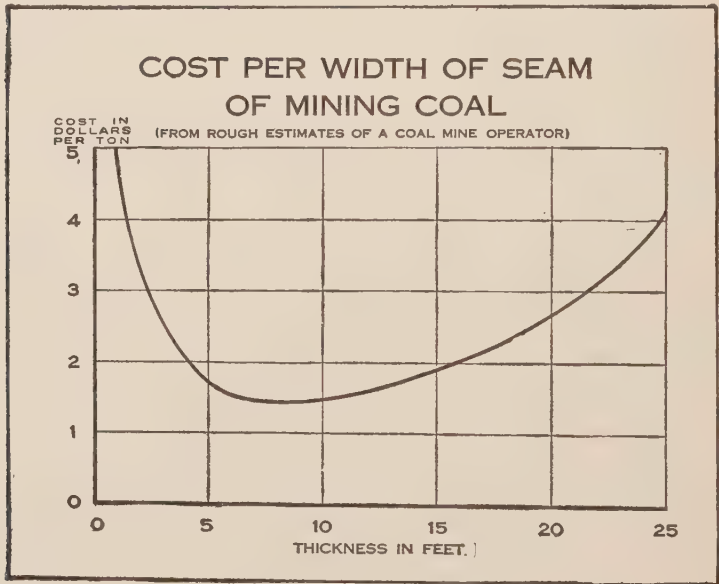


FIG. 205.—Primary correlation.

Primary Correlation.—Primary correlations are studied as between two variables which are compared by being grouped together according to their magnitudes. Thus, in Fig. 205, the thicknesses of the different seams of coal are measured on the horizontal scale, while the costs of mining these different thicknesses are measured on the vertical scale, and the relation of the

costs of mining to thicknesses of seams is shown by the resulting curve. Since the thickness of the seam is the standard by which we consider costs, it is the "independent variable" and is measured by the horizontal scale. Depending upon the thickness of the seams, the cost of mining will vary and this, the "dependent variable," is measured by the vertical scale.¹

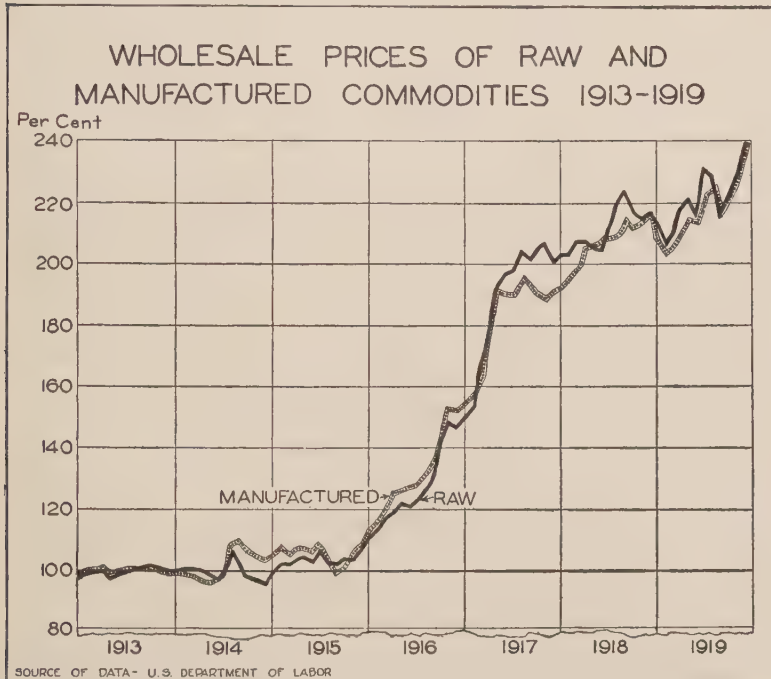


FIG. 206.—Secondary correlation between two time series.

Usually a correlation chart of this type is made as a curve but occasionally it is made as a vertical bar chart when the series is not continuous.

Secondary Correlation.—The graphic method is often used to show the correlation of two variables by plotting each against

¹ By convention, the "independent variable" is plotted on the horizontal scale and the "dependent variable" is plotted on the vertical scale in all statistical charts. The horizontal scale defines the questions and the vertical shows the answer. One determines the independent variable when he decides upon the terms of his problem. The readers of charts like Fig. 205, for instance, will recognize thickness of the seams as the basis of the study, but will fix their immediate attention upon the dependent variable or costs of mining, measured upon the vertical scale.

a common third variable, as is shown in Fig. 206. Thus, in this chart, raw commodity prices are plotted as ordinates with years as abscissas, and also manufactured goods prices are plotted as ordinates with years as abscissas. Since in general the contours of the two curves are alike, it may be said that there is correlation between the two. However, care must be used when studying correlation by this method. While there may be a close correspondence between the two curves, there may be no causal relation whatever, and hence no correlation. Although curves drawn in this way may appear to correspond closely except for occasional differences, yet these few differences may mean a great variation in the degree of correlation.¹

Historical Time Series and Frequency Series Distinguished from other Correlative Series.—In a general sense any graph with horizontal and vertical scales, in which the corresponding values of two variables are represented by a single point, is a correlation curve. However, since in business statistics historical graphs covering periods of definite dates are used probably more than all other kinds combined, they have been treated separately in Chaps. V and VI. It should be remembered that a *historical* graph is a graph of *dates*. Not only is a period of time expressed along the horizontal axis, but this period represents definite dates, such as certain months or years. In the correlation chart shown in Fig. 207, a period of time is represented by the horizontal scale, and in Fig. 212 (lower part) time is represented by the vertical scale, but these are not historical graphs since the time periods are not definite dates. Only the lengths of the periods are important in these charts and dates are not significant.

Another important kind of graph, the frequency graph, has been considered separately in the preceding chapter (Chap. VII). The frequency graph shows for each class the *number* of occurrences. The correlation curve shows the effect of the

¹ The degree of correlation may be determined mathematically by means of the coefficient of correlation. The student should be warned, however, against spurious correlation. No mathematical calculation is complete without an estimate of the probable error in the coefficient of correlation. On the whole, carefully handled and interpreted graphics have some advantages over mathematical correlation. The elements of judgment and special knowledge of conditions can be introduced in reading graphical correlation. The mathematical calculation, on the other hand, is very rigid. For further information on the coefficient of correlation see G. U. YULE, "Introduction to Statistics," or any standard text on statistics.

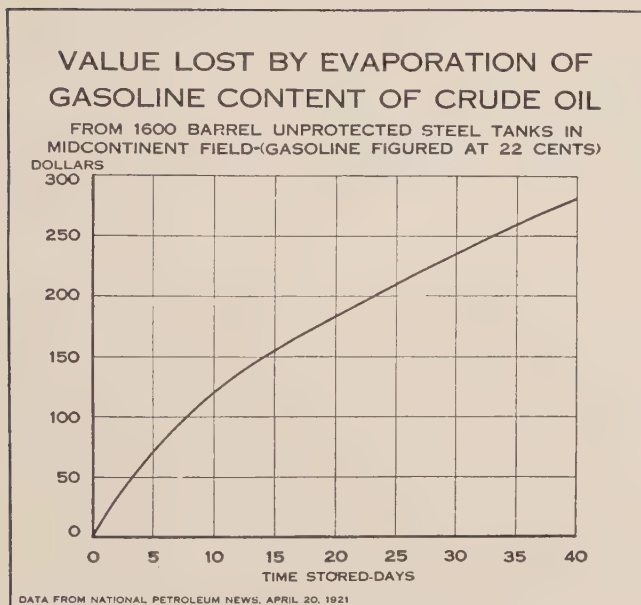


FIG. 207.—A time series but not historical.

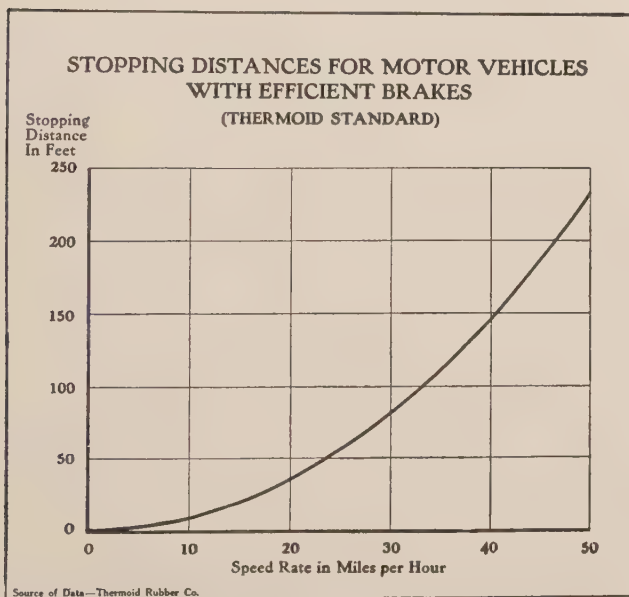


FIG. 208.—When showing a continuous series the curve should be smoothed.

independent variable upon the dependent variable or, for instance, the effect of the width of seams upon the cost of mining coal (Fig. 205).

Smoothed and Broken Curves.—If the nature of the data is such that changes in the curve are gradual, with possible values at all points on the curve, the smoothed curve should be used (Fig. 208). Thus, in this figure there is a stopping distance for

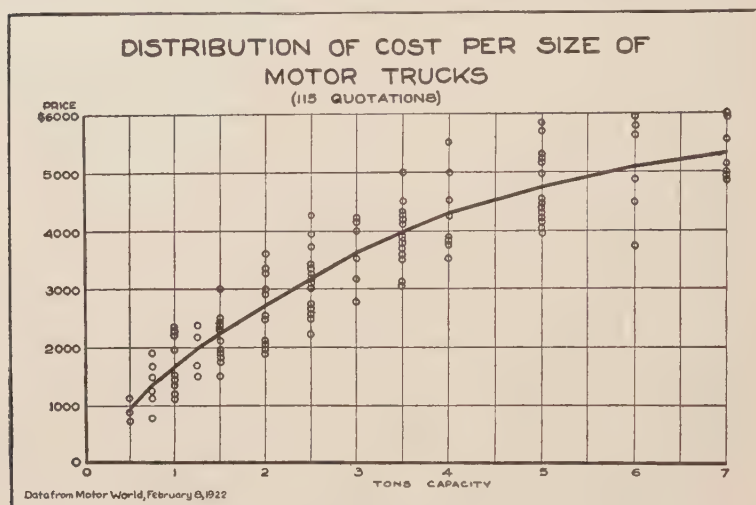


FIG. 209.—When plotting a broken series, the points should be connected with straight lines.

every possible speed rate up to 50 miles per hour, and as the speed rate gradually increases, the stopping distance gradually becomes longer.

If, on the other hand, there cannot be values for every point, the broken curve should be used (Fig. 209). Here the plotted points represent values, but all the connecting lines do is to point the direction from one value to another, thus aiding the reader in getting a general impression of the distribution. There cannot be values between points in Fig. 209, since motor trucks, according to the sample, are made only in the sizes designated. If a bar chart is ever used in correlation charting, it should be used for the kind of distribution shown in Fig. 209 rather than for the kind shown in Fig. 208.

Position of the Curve.—In fitting a curve to a large number of scattered points (Figs. 209 and 210), care must be taken to have

each portion of the curve as nearly as possible at the center of gravity of the dots in any vertical section of the chart.¹ A curve may not be correctly placed by having the same number of dots on each side, for their distance from the curve must be considered. As a simple rule to be kept in mind when fitting a curve, one may consider the dots in any vertical section of the graph as weights arranged on a horizontal lever, with the curve as the fulcrum, and then shift the curve so that the lever will balance (Figs. 209 and 210).

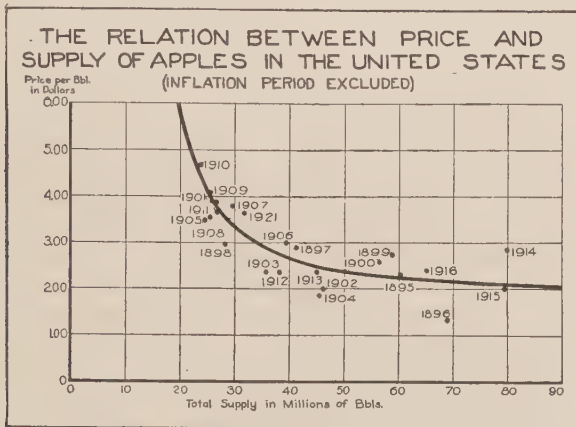


FIG. 210.—Fitting a curve to scattered points.

Comparison of Correlation Curves.—Figures 211 and 212 are examples of the comparison of correlations. Figure 211 shows the differences in the costs of the gas and electric arc methods for cutting steel for any thickness within the extremes of the chart. Figure 212 compares the differences in costs and time used, in relation to distance, in transportation by railroads and motor trucks.

When comparisons such as Figs. 211 and 212 are made, the units of the scales must be common to all of the series that are plotted. That is, for instance, in Fig. 211 if thicknesses for the gas method are measured in inches, then thicknesses for the electric arc method must be measured in inches, and if the cost for gas is measured in cents per foot of steel, then the cost of electricity must be measured in the same way.

¹ If the points fall close to a curve, there is said to be a high degree of correlation. If they are widely scattered, it is probable that outside factors have a strong influence and correlation is said to be low.

Note that the points are widely scattered in the "Freight" curve of the lower part of Fig. 212, and hence the degree of correlation is very low. When curves are freely smoothed, the actual plotted points should be indicated clearly as shown in this chart (Fig. 212).

These charts (Figs. 211 and 212) were drawn on arithmetic vertical scales because they were intended to show the differences

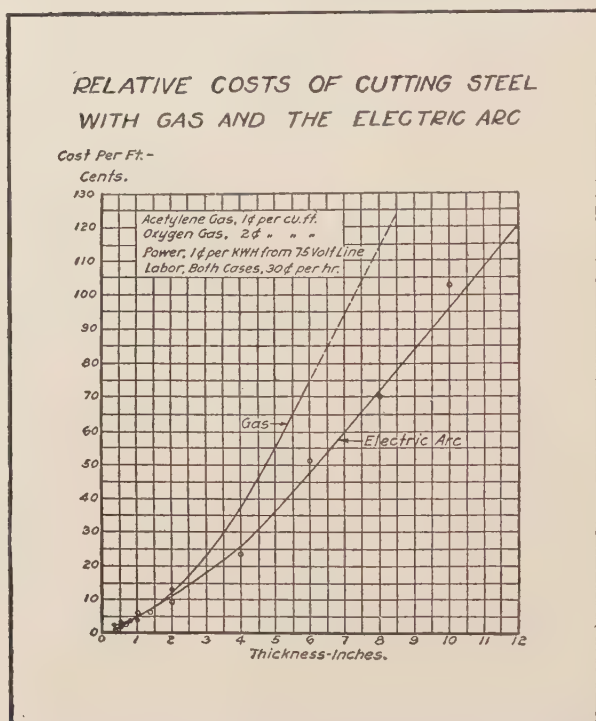


FIG. 211.—Comparison of correlation curves. (Courtesy of Industrial Management.)

in the sizes of the costs and time periods involved. If, however, not the *differences* but the *rates of change* are to be compared, logarithmic scales should be used.

Use of the Logarithmic Scale.—Figure 213 shows how an investment of \$1 would increase at 4, 5, and 6 per cent compound interest for 40 years. Part A of Fig. 213 shows the curve plotted on an arithmetic scale, and its purpose is to show the size of the investment for any particular length of period under 40 years.

It does not show the rates of increase, and it does not indicate that the rates of increase are more rapid toward the end of the period. The increase rates are constant, and this is shown by the straight lines on part B of Fig. 213, which is plotted on a logarithmic scale.

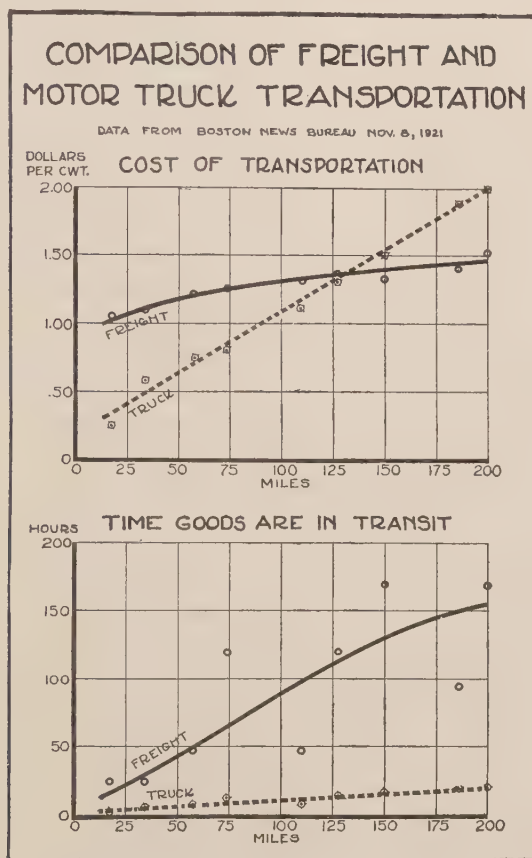


Fig. 212.—Comparison of correlation curves.

In reading the time chart in Fig. 212 (lower chart), it must be understood that the purpose is to show differences and not ratios. The rates of change in the two curves as the mileage increases cannot be directly compared. In attempting to do this, one is likely to decide that, as the distance increases, the time taken by freight increases at a faster rate than that taken by truck

transportation. But this is not the case as is readily seen by plotting the curves on a logarithmic vertical scale as shown in

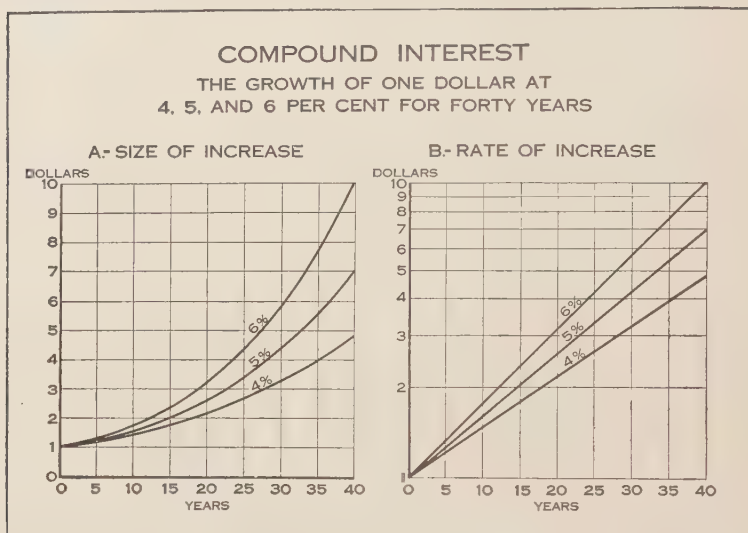


FIG. 213.—Arithmetic and logarithmic scales. (Time series but not historical.)

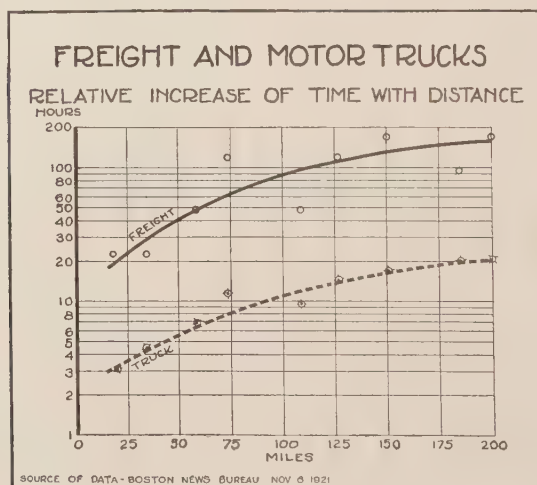


FIG. 214.—Logarithmic correlation curves.

Fig. 214. As the logarithmic chart is fully described in Chap. VI, it will not be considered further at this point.

CHAPTER IX

GEOGRAPHICAL DISTRIBUTIONS

When it is desired to show the relations between magnitudes, frequencies, or ratios and their geographical distributions, various kinds of statistical maps are used. The chief function of statistical maps is to show graphically the positions of the facts represented. This cannot be done by bar charts and curves, for the order of arrangement cannot show directly what items are contiguous or remote in their spatial distribution. Not only magnitudes, frequencies, and ratios, but positions are visualized on statistical maps.

Outline Maps.—Plain outline maps with but little or no coloring are by far the best for statistical work.¹ Mountains, streams, forests, cities, state names, and the like, should not be shown unless they add a distinct value to the presentation.

Simple Comparisons of Geographical Areas and Distances.—Occasionally it is desirable to compare geographic magnitudes in connection with showing maps of the areas. This is done in various ways simply by using maps which are drawn to a common scale. This usage is illustrated by Fig. 215, which compares the length of California with other parts of the United States in a more vivid way for the reader of popular publications than would the ordinary bar chart. The principles involved in making this kind of comparison have been considered at length in Chap. IV.

Simple Distribution of Size.—The most simple type of statistical map is that which represents sizes in their geographic positions. It should be noted that Figs. 216 to 219 inclusive represent magnitudes for points rather than for areas, and that the maps show the distribution of these points. In Figs. 220 and 221 the objects are not to show sizes at points but to show distances between points. In Figs. 223 to 229 inclusive the sizes are represented not for points but for certain defined areas.

¹ Satisfactory maps can usually be obtained from Rand McNally and Co., Chicago; National Map Co., Indianapolis; Educational Exhibition Co., Providence; U. S. Geological Survey, Washington; State Highway Commissions; City Engineers; and other government offices.



FIG. 215.—Simple comparison of geographic distances.

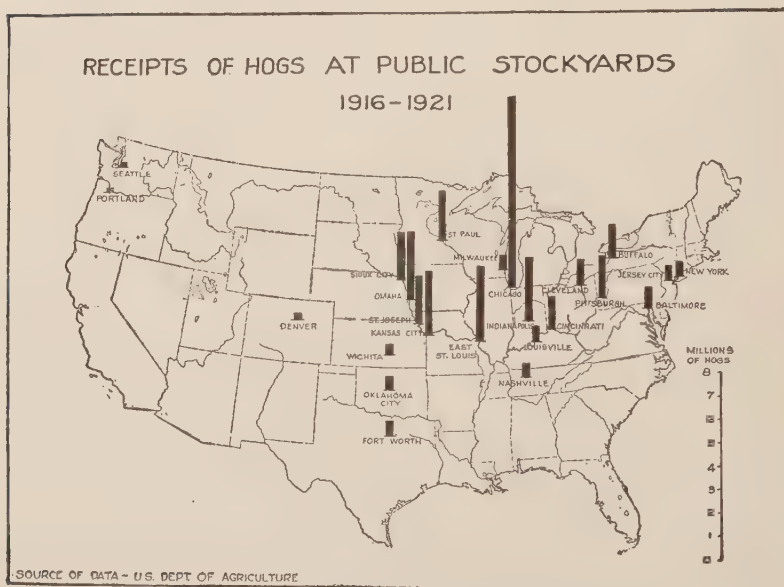


FIG. 216.—The use of bars to show sizes at geographic points.

Methods of Representing Numbers.—Lines or bars, circles or dots, and cross-hatched shades are the most common devices used to express size on statistical maps. These are used in a large number of ways of which only the most common will be discussed here.

Magnitudes at Geographical Points.—For graphically showing simple sizes, bars are sometimes used as illustrated in Fig. 216, in which the length of the bar is proportional to the size repre-

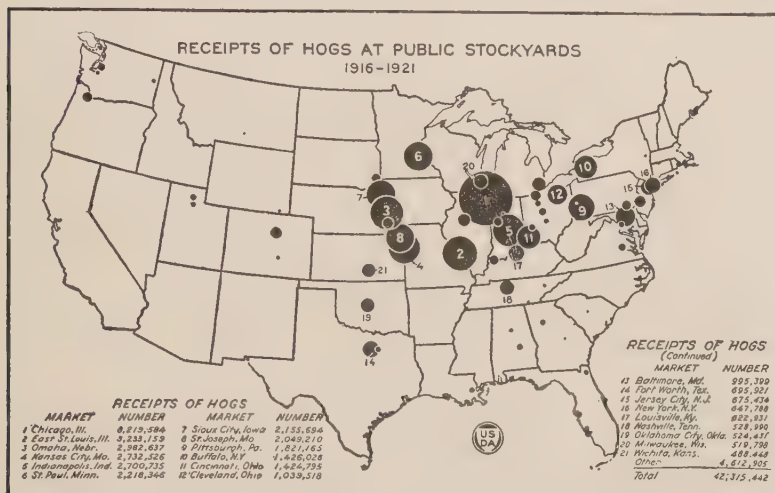


FIG. 217.—The use of circular areas to show sizes at geographic points. (Courtesy of United States Department of Agriculture.)

sented. This shows directly at a glance much more regarding distribution than would the ordinary bar chart, but sizes cannot be compared as easily since the bases of the bars are not in a straight line. When using this method, all of the bars should be drawn to the same scale, and it is well to include the scale as shown in Fig. 216 as an aid in making careful comparisons.

The geometric form most commonly used on this type of map is the circle or dot. Either the circle or circular area is used, depending upon the distribution of the data. Circular areas or dots show up better when there is room for them (Fig. 217), but if they overlap, circles are most clearly read (Fig. 218). In both cases the circles should be drawn so that their areas are proportional to the sizes represented. If a note to the effect that they are so drawn is included on the chart, the reader will know that they are drawn on an area basis and not on the more misleading

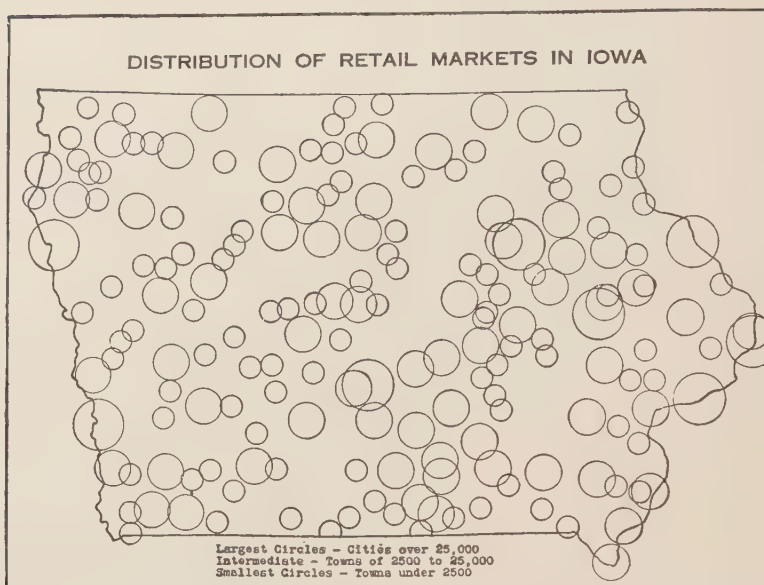


FIG. 218.—The use of circles to show sizes at geographic points. (*Adapted from Woman's World.*)

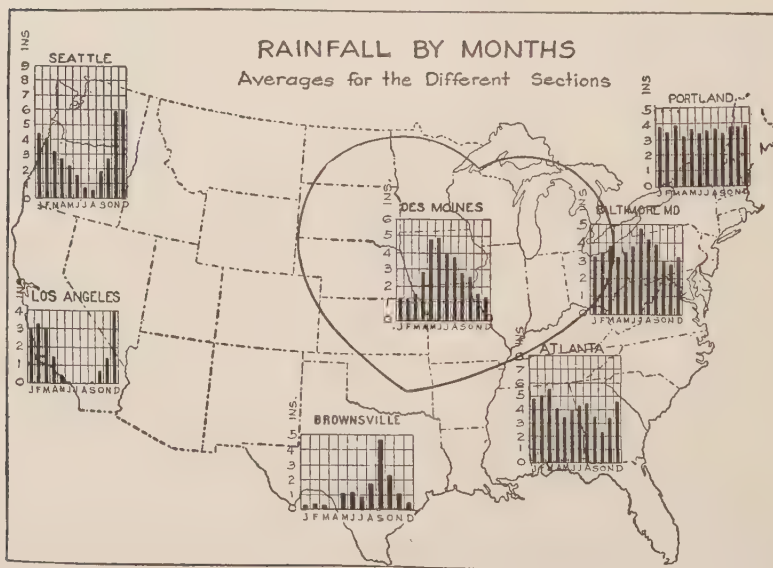


FIG. 219.—Geographic distribution of time variables. (*Courtesy of Successful Farming.*)

diameter basis which is sometimes used. In a map like Fig. 218, the limits of the class intervals should be stated definitely and the circle should be drawn in proportion to the middle of the class. It should be noted that in this respect this map is faulty. (Other reasons entered into the determination of circle sizes in the original from which Fig. 218 is adapted.)

Figure 219 illustrates a good method of showing a *series* of observations at each point. These little time charts can be made

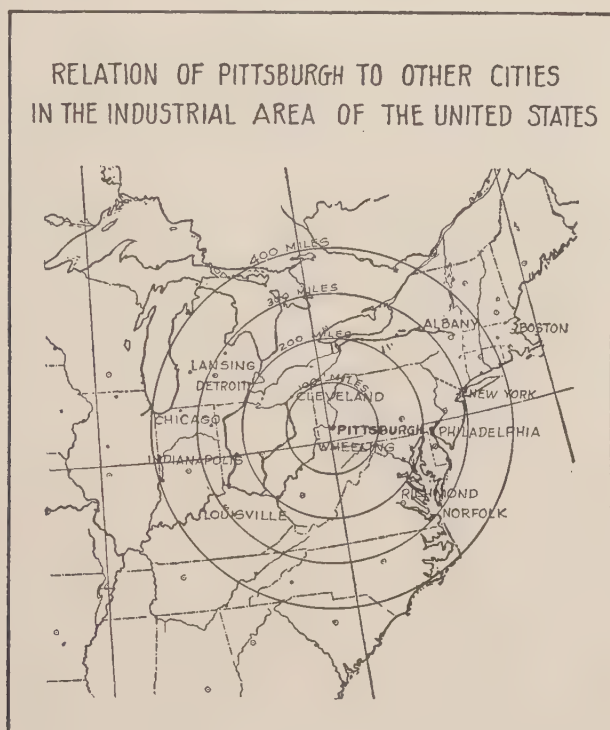


FIG. 220.—The use of circles to show distances from a central point.

also in the curve form on such maps. Figure 219 would be improved, however, by using the same scale intervals and the same range for all of the graphs.

Radial Distances.—Maps comparing radial distances differ from the foregoing maps as only the distance between the center and the circle is of significance (Fig. 220). Such maps are made

most commonly as shown in Fig. 220, but they are greatly improved by using the method of shading illustrated in Fig. 221.

So far we have considered only sizes at points or distances between points. Several other factors must be considered when showing distributions of size in relation to areas.



FIG. 221.—The use of shading makes a clearer presentation than the method used in Fig. 220. (Courtesy of St. Louis Chamber of Commerce.)

Representing Sizes for Certain Areas.—The graphic forms described for representing magnitudes at points can be used also for representing magnitudes for areas, though often the effect is only of doubtful value when used with areas varying in size. In addition, the method of cross-hatching or shading, in which the intensity is proportional to the number, can be used.

Statistical maps of this kind compare data which have been collected on an area basis. These areas may be equal, such as square miles, or they may be unequal, such as states or counties. Often in making a map of the United States it is impractical or even impossible to map the data on the basis of units smaller than states. If, for instance, the population figures of the different states are compared, unequal areas are compared also. That is, in comparing the sizes of the populations of Massachusetts and Missouri, we find that of Massachusetts approximately

one-eighth larger. However, this may mean nothing as the area of Missouri is about eight and one-half times that of Massachusetts, and the number of persons per square mile in Massachusetts is nearly ten times that of Missouri. It is often desired to show simply the number of sizes by unequal areas such as states, with little regard for densities of the distributions. That is, we may wish to compare the number of automobiles without showing how close together they are. Figures 223, 226, 227, 228, and 229 illustrate methods of showing such data.

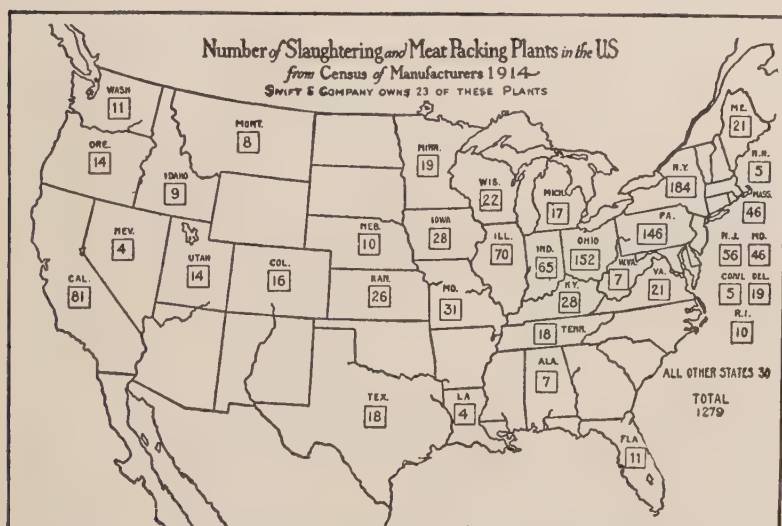


FIG. 222.—No direct visual impression of magnitudes is left by this map. Compare with Figs. 223, 226 or 227. (Courtesy of Swift & Co.)

In maps like Fig. 223, the dots should be drawn with their *areas* proportional to the numbers represented, and it is well to include a note to this effect on the map in order that the reader may know that they are not drawn on a diameter basis. It is also well to include a scale for reference as illustrated in Fig. 223. For more exact reference, the actual numbers may be placed in the corners of the states in small figures. However, figures alone leave no direct visual impression of the magnitudes, and maps like Fig. 222 show merely the geographical distribution of the figures of a table. Some kind of bar, dot, shading, or other common graphic device should be used when it is desired to give a visual impression of the relations. Figure 224 shows the dot method of Fig. 223 used on a county basis.

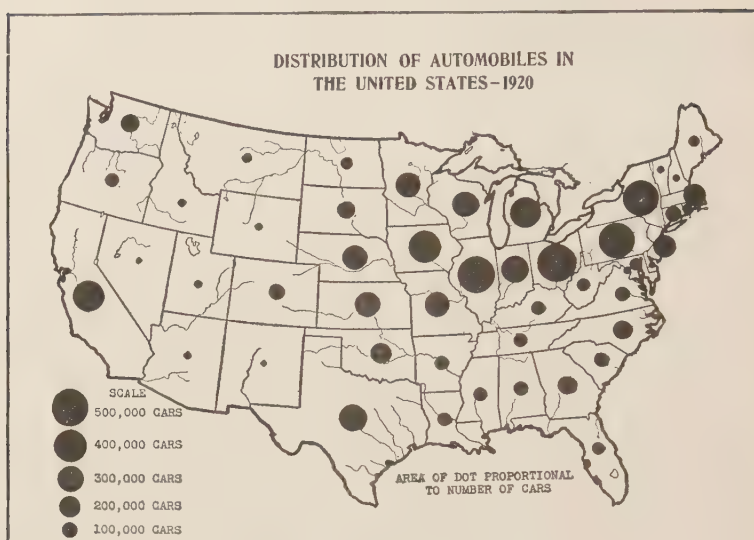


FIG. 223.—Use of circular areas to show sizes.

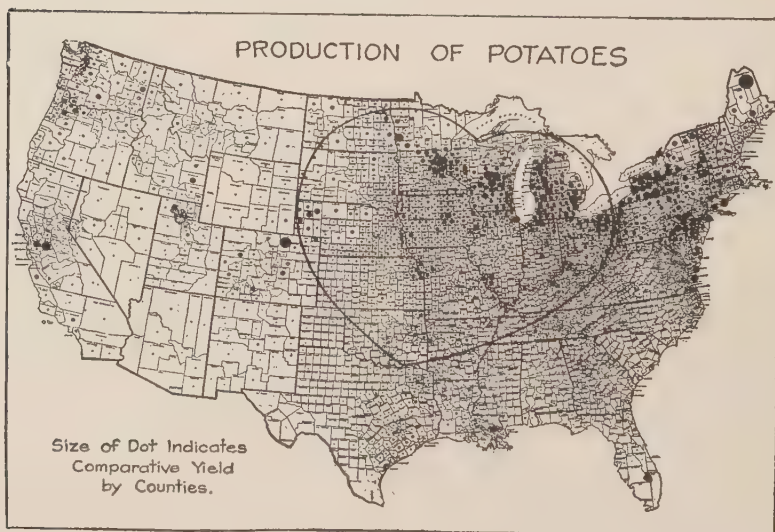


FIG. 224.—Use of the method shown in Fig. 223 on a county basis. (Courtesy of *Successful Farming*.)

A map on which the sizes of the dots vary, gives a good general impression very quickly and easily but it is likely to leave false impressions if an attempt is made to read it accurately. This is true because circular areas are difficult to compare. To most persons a circular area twice the size of another appears to be less than twice as large.¹

This objection is also true of Fig. 225, but since each dot is a small pie chart showing the used and unused parts of the whole, it has an additional value. While the sizes of the fields are somewhat difficult to compare, the percentage of exhaustion is very clearly shown.

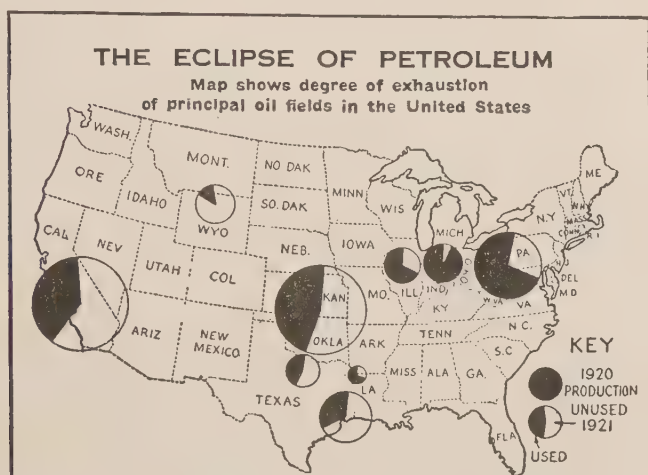


FIG. 225.—Geographical distribution of aggregates showing their component parts. (Courtesy of Automotive Industries.)

Another type of dot map is illustrated by Fig. 226. Instead of using different-sized dots to indicate different values, uniform sizes are used, and the number of dots for the different states is proportional to the number of cars represented. Thus, in Fig. 226 one dot equals 50,000 automobiles. Note that the numbers are represented to the nearest 12,500 by using the quarter of a dot. When this type of map is made, the purpose is to represent not *densities* but *sizes*. Thus, the unit which one dot is to represent should be chosen carefully in order that in no case will there be so many dots that they cannot be counted at a glance, and the arrangement of the dots should be such that the groups in the

¹ See p. 44.

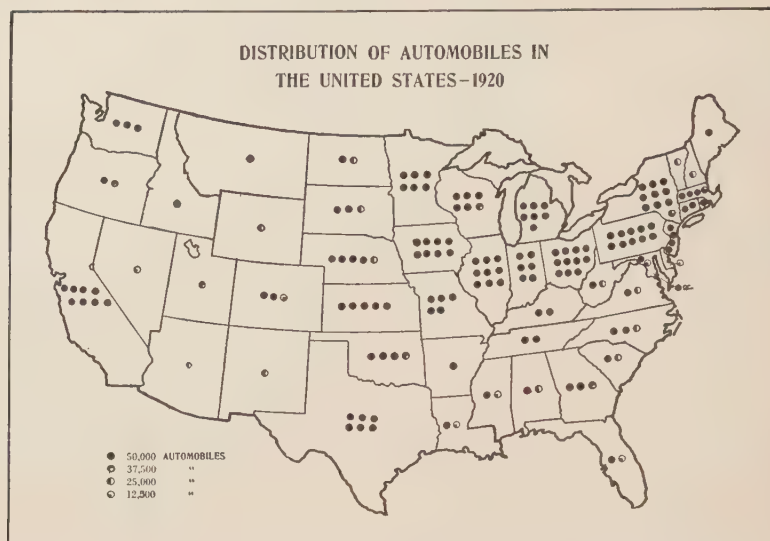


FIG. 226.—Grouping dots to show sizes. (This type of map should not be confused with Fig. 232, in which the primary purpose is to show density instead of size.)

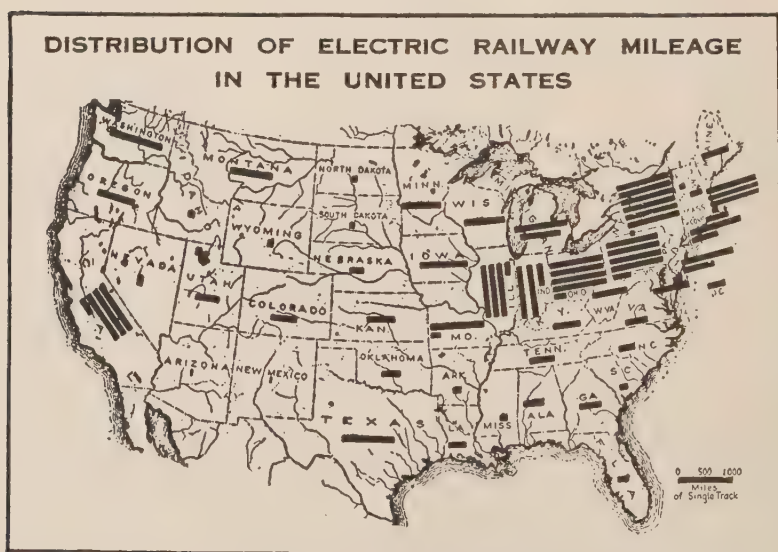


FIG. 227.—Use of bars in showing geographical distributions of sizes. (Courtesy of the ABP News.)

different states may be compared easily. They should be arranged squarely in straight rows so that the number of dots can be seen at a glance, and they should be equally spaced in all states so that the spaces covered by the dots will not give inaccurate impressions. Thus, ten dots spread over a larger space than that covered by twelve dots might impress the reader as representing the larger number. This method should not be confused with the density dot maps discussed on pages 195 to 200.

The method illustrated in Fig. 227 is similar to that of Fig. 226 except that instead of a row or group of dots, the length of

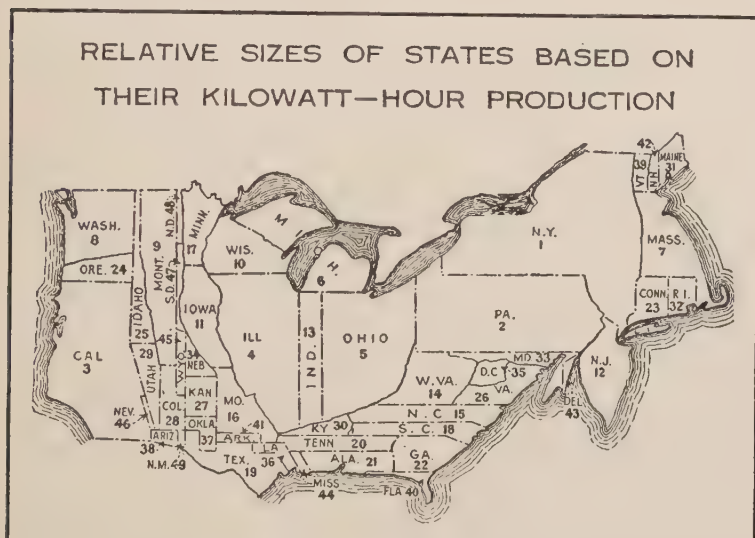


FIG. 228.—States drawn in proportion to the production figures represented. This is not a good method for general use though occasionally it is quite effective. (Courtesy of Mechanical Engineering.)

the bar represents the numbers. The scale to which the bars are drawn should be selected carefully. It should be small enough to allow the sizes of the states on the maps to include the bars, and large enough to show all *significant* figures. This scale should always be shown for reference (see Fig. 227).

Figure 228 represents a method of showing values for states by drawing the area of each state on the map proportional to the number represented. Notwithstanding the fact that these maps are much in vogue at the present time, it is not evident that they are superior to the dot or bar maps just described (Figs. 223 to

227 inclusive). The states are so distorted that little resemblance remains and even the relative positions are inaccurate. It is much more difficult to compare their irregular areas than it is to compare either dots or bars.

Representing Size by Shading.—Different shades of cross-hatching may be used to represent magnitudes. Thus, in Fig. 229 the coal supply in number of days is represented for each state by different densities of shading.

In the cross-hatched map, the shading ranges from white to black—the extreme shades illustrating the extremes of the thing represented, and the middle shades that which is the more com-

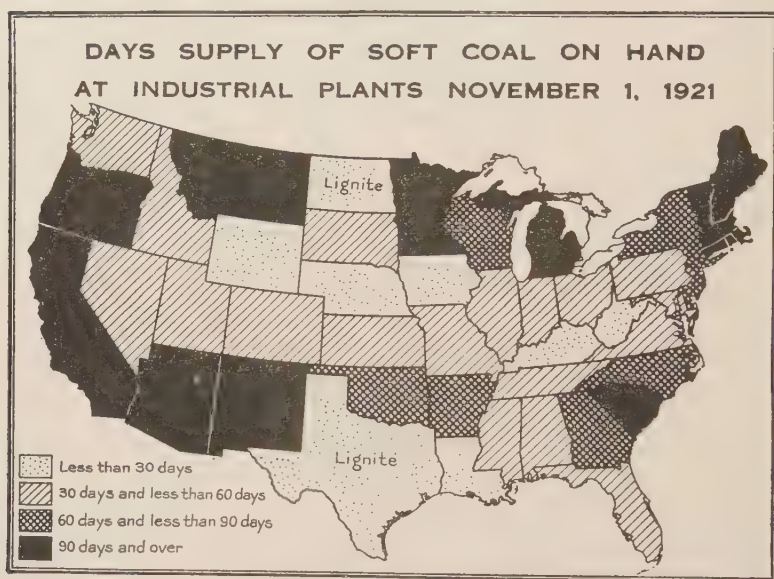


FIG. 229.—Representing sizes by shading. (Courtesy of the United States Geological Survey.)

mon. The number of shades to be used depends upon the number of classes into which the data are divided, but in nearly all cases the light shading should represent the small extreme and the dark shading the large extreme. These classes should be of uniform size, and the shades used to represent them should vary to the same degree. In Fig. 229, four different kinds of shading varying from light to dark are used. A clearer impression would be given if shades differed by the same degrees of "blackness." That is, the lowest shading might be 0 per cent black, the second,

$33\frac{1}{3}$ per cent black, the third, $66\frac{2}{3}$ per cent black, and the highest, 100 per cent black.

Since the differences in the scale are represented by differences in the proportion of area covered by ink, it is best to make the changes as simply as possible. An elaborate design of shading cannot be understood as easily as the simple form illustrated in

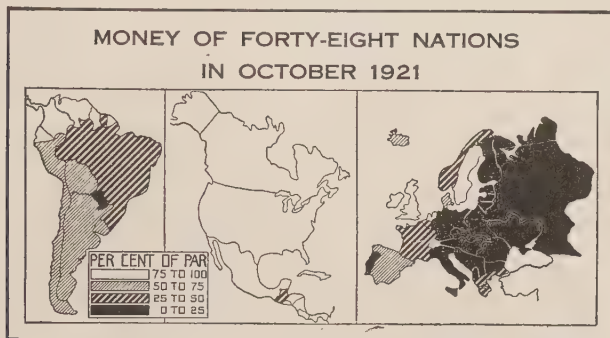


Fig. 230.—A simple method of shading to show percentages. (Courtesy of the Cleveland Trust Co.)

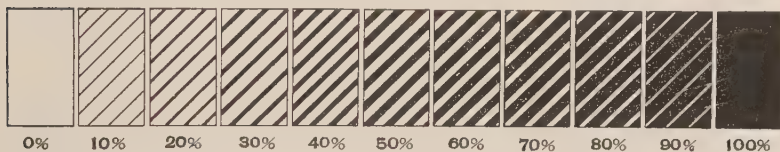


Fig. 231.—A simple method of shading mathematically graduated from 0 to 100

Figs. 230 and 241 in which even the direction of the lines does not change. Any number of shades can be made very easily, and with their differences mathematically exact, by using the method of cross-hatching illustrated in Fig. 231, in which the shades are shown with a variation of 10 per cent from zero to one hundred.¹

¹ This shading is very simply and quickly made with the aid of a section liner. Simply determine the width of line to be used and adjust the section liner so that it will move exactly this width. Then, in making a 10 per cent shade, for instance, draw a line, then move the section liner nine times for the white part, then move again for another black line, and so on. For a 50 per cent shading draw five lines together, then move the section liner arm five spaces for the white part, and so on. For a 90 per cent shading, draw nine lines together and skip one, and so on.

If a large amount of map work is to be done for reproduction by a photographic process a great deal of labor can be saved by drawing the shading on sheets and making photographic copies. If a photostat is used, only the

Usually it is not practical to have as many shades as are shown in Fig. 231, but it will be noted that the values represented can be very quickly appreciated even without reference to the scale. It is not difficult to see, for instance, that 10 per cent of the area is black in the 10 per cent shade, or that half of the area is black and half of it is white in the 50 per cent shade.

Such maps as Figs. 229 and 230 give the impression that absolute uniformity prevails within each division and that changes occur only at their borders. While this would be true, practically speaking, of Fig. 230, it would not be true of Fig. 229 where the changes naturally occur gradually from one district to another.

The types of maps illustrated in Figs. 223 to 229 inclusive are widely used, but often they are of questionable value and sometimes it is impossible to make practical comparisons from them. The chief difficulty is that areas vary as well as the numbers represented. In studying spatial distribution, it is often quite necessary to have the values represented on the basis of a common unit such as per square mile. It means but little, as far as area distributions are concerned, to say that there are nearly twice as many automobiles in Iowa as in Massachusetts (Fig. 226), unless we know that Iowa is approximately seven times as large as

lower half of a series, as in Fig. 231, need be drawn, for negatives of these will make positives of the upper half. These sheets are pasted on the map in their proper positions.

In mounting these shades, only the edges adjoining the finished part need be trimmed to fit. Thus, assume that we are pasting shades on a map to make Fig. 241. Suppose that we start with the state of Washington. We will cut the north and west sides to fit and let any surplus project over Idaho and Oregon. Then, when we mount the shade for Oregon, we will trim the west and north sides properly and mount it in its proper position, covering the surplus shade of Washington and allowing the surplus shade of Oregon to extend over the unshaded states to the south and east. Then, when we mount the shade for Idaho with its western side correctly shaped, it will cover the surplus of Washington and Oregon and give their eastern boundaries their correct shapes. This process can be carried on until the last states are shaded and these will have to be trimmed all around before mounting. If, however, one prefers to do so, in a case of this kind the shades may be allowed to extend over the national borders, and, when completed, the whole map may be trimmed and remounted.

For the sake of simplicity all lines should be made in the same direction. The lines should be drawn at an angle of about 45 degrees with the border line of the map in order that the changes may not be noticed in the angles that these lines form with the meridians and parallels of latitude.

Massachusetts, and while we can see easily that there are eight dots in Iowa and three and three-fourths dots in Massachusetts, it is very difficult to see that Iowa has approximately seven times the area of Massachusetts. In Fig. 223 it is not possible to see easily that the total area of the dot in Iowa is approximately twice the area of the dot in Massachusetts. For most purposes the most satisfactory map is the density map which will now be considered.

Densities.—In studying area distribution, usually the densities are the important facts. The important questions are:

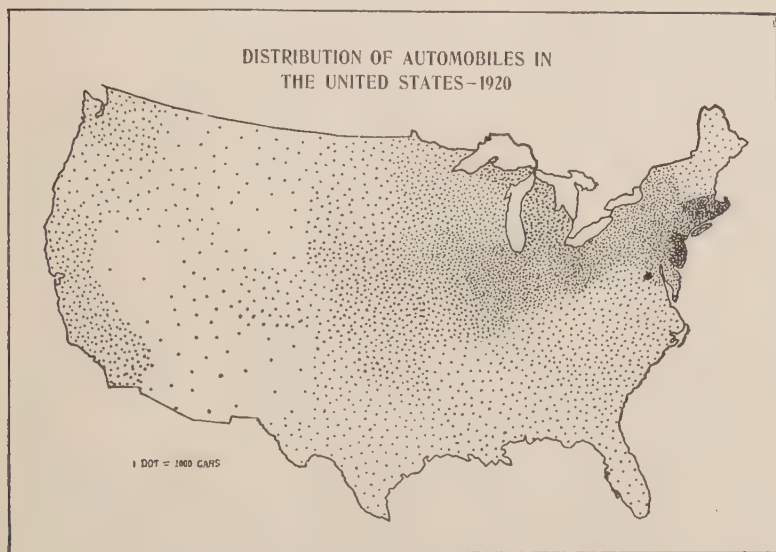


FIG. 232.—The use of dots for indicating density.

Where are the points showing a high degree of concentration? or, Where are the sparse distributions? In showing distributions where area is a factor, the density map made with dots, as illustrated in Fig. 232, is commonly used. In this type of map all dots are of the same size and the distributions are shown by the frequency with which these dots occur. Numbers are not well shown by this kind of map but the relative densities are clearly shown. Such a value should be assigned to the dot as will best fit in between the extremes. That is, the value should not be so small that there would not be room for the dots in the

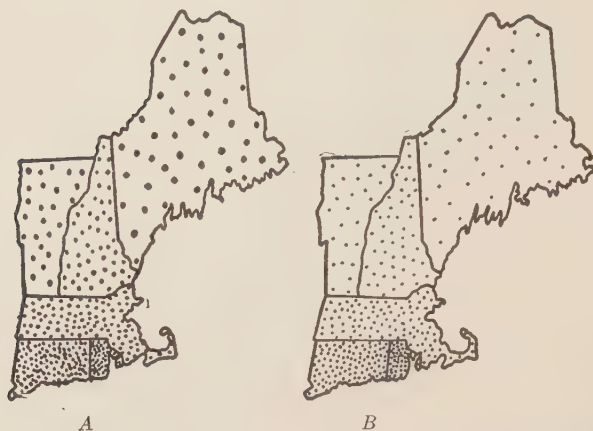


FIG. 233.—*A*, Wrong because the larger dots in the sparse areas exaggerate the data. *B*, Right because density is correctly shown by dots of equal size.

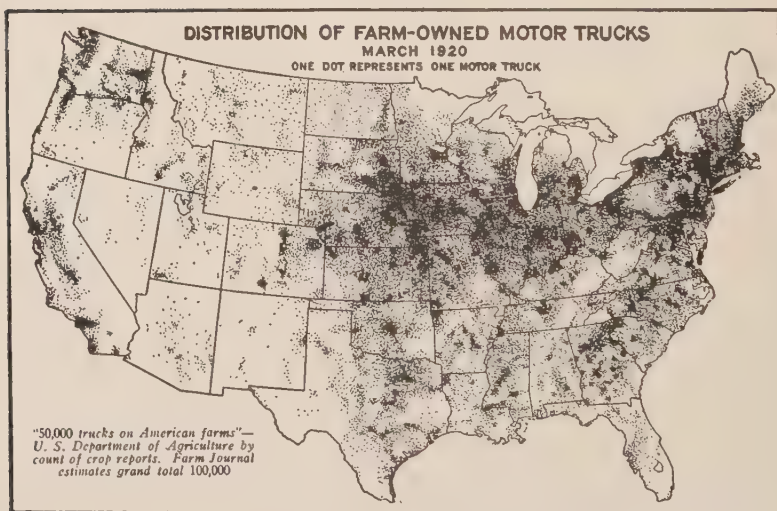


FIG. 234.—Density map of the United States drawn on a county basis. (Courtesy of the National Automobile Chamber of Commerce.)

concentrated areas—they may run together but there should not be several layers—while, on the other hand, the unit should not be so large that important data in the sparse areas would be incorrectly represented or omitted.

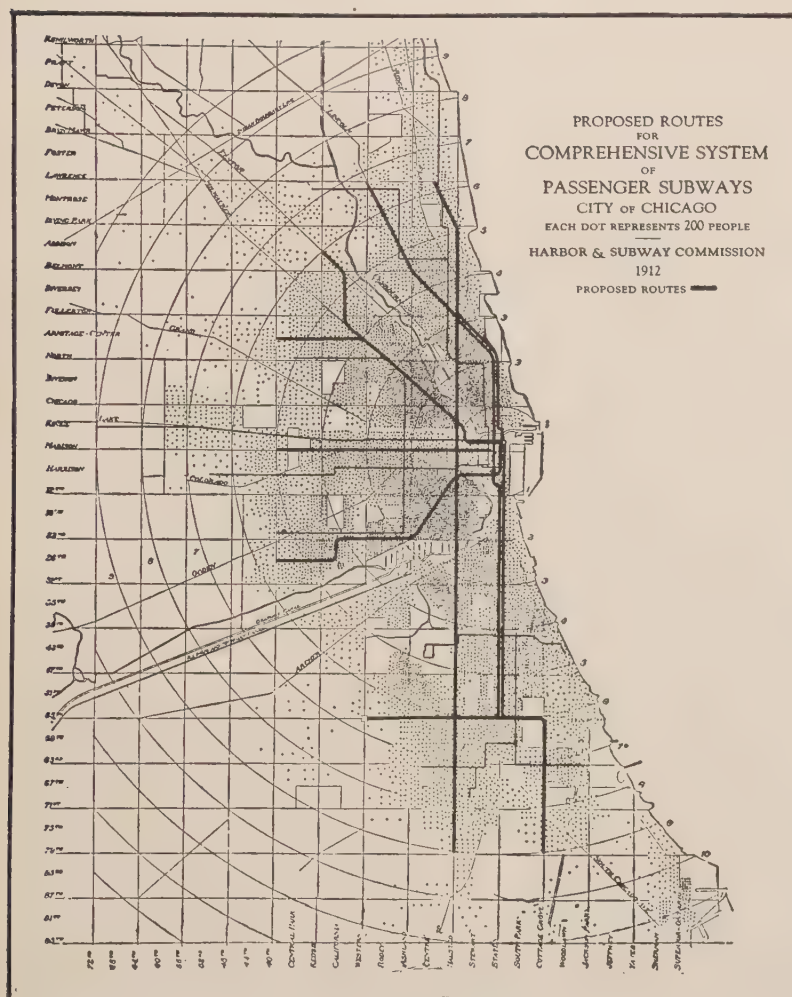


FIG. 235.—Density map with dots representing 200 persons on a center of gravity basis. (Courtesy of the Department of Public Works, City of Chicago.)

State lines are sometimes omitted as in Fig. 232 but more often they are left in as a guide to the reader. The dots should be spread evenly over the area and care should be taken to see

that they are not placed in rows.¹ It is important to use dots of uniform size in a density map. Otherwise the effect will be as illustrated in Fig. 233.

Since the areas of the states on the map are proportional to their actual areas, the dots are automatically distributed on a square mile basis if they are spread evenly over each state. Maps like Fig. 232, of course, show the average distribution for each state, and so do not show the concentration points at large cities.

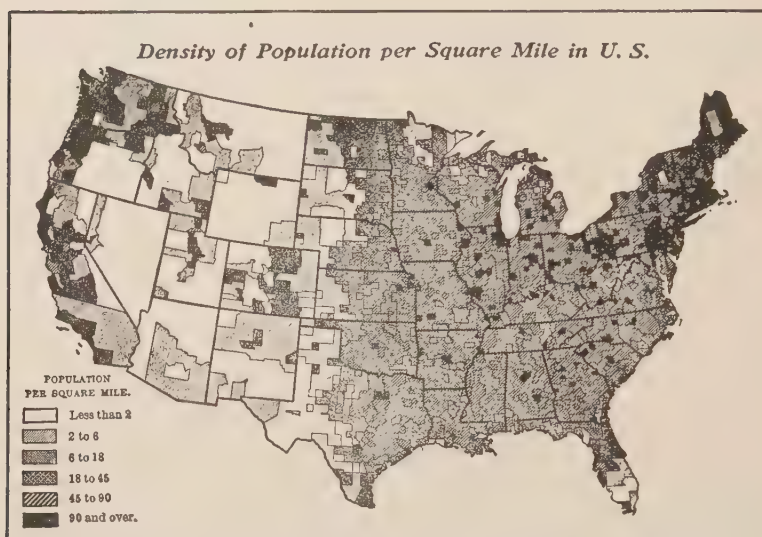


FIG. 236.—In showing density the use of dots is usually to be preferred to the use of shading. When shading is used, however, the class intervals should practically always be equal.

The concentration points show up better when the map is made on the basis of small divisions such as counties in a map of the United States (Fig. 234), or townships in a map of a state, or blocks in a map of a city. The degree of accuracy necessary for maps of this kind depends upon the purpose for which the map

¹ After the unit has been selected, the number of dots for each state or division may be determined and the figures penciled on each area. Then, with a single stroke of the pen for each dot, and counting them as you go, keep working over the whole area, placing a dot where the largest space remains until the required number is placed. The Payzant pen is good for this work since it makes a perfectly round dot.

is to be used. In Fig. 235 each dot is carefully located to represent 200 persons. The dots here are not spread evenly over any particular area but are supposed to be located near the center of gravity of each group of 200 persons.

The type of map illustrated in Figs. 232, 234, and 235 gives the impression of gradual changes from dense to sparse distributions, and seems to smooth out the breaks between political jurisdictions, which are especially noticeable when shading is used as in Fig. 236. It is not necessary to use the method of

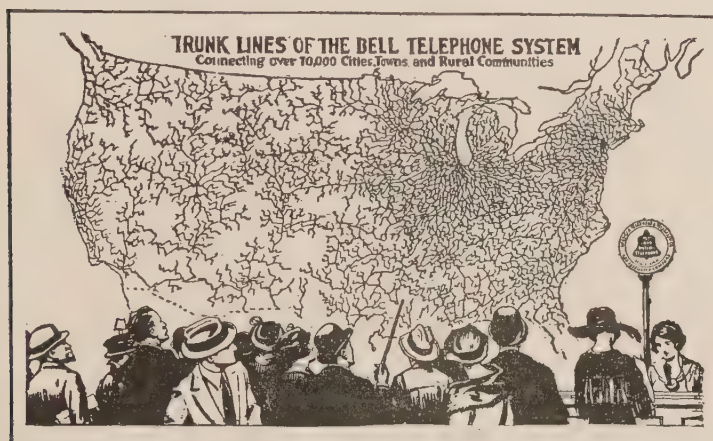


FIG. 237.—Density of linear magnitudes. (Courtesy of American Telephone and Telegraph Co.)

cross-hatching in maps like Fig. 236 where area is a factor. The actual areas are proportional to the areas on the map and the density can be appreciated better by using a dot map like Fig. 234. Note that in Fig. 236 the class intervals vary in size from two in the lowest group to forty-five in the next to the highest group, with no upper limit in the highest group. This makes the map particularly difficult to comprehend clearly. Class intervals should be equal in size on maps of this kind, though occasionally there may be practical exceptions to this rule. The difficulty, of course, is to show the sparsely populated areas accurately. This would be much easier to do by using a dot density map such as Fig. 234.

While dots are good for representing separate objects, such as automobiles, factories, persons, and the like, they are not truly representative of the density of linear magnitudes, such as roads,

railways, and telephone lines. This kind of density distribution is very clearly shown by using a map like Fig. 237 which shows the density of Bell Telephone trunk lines simply by mapping the lines in their actual positions. The concentration area of Swift and Company's shipments is well shown by the method illustrated in Fig. 238.

In Fig. 239 the density of street railway traffic is shown by the widths of the lines. This is the common form used for showing traffic flow. The widths of the lines are laid out to a common

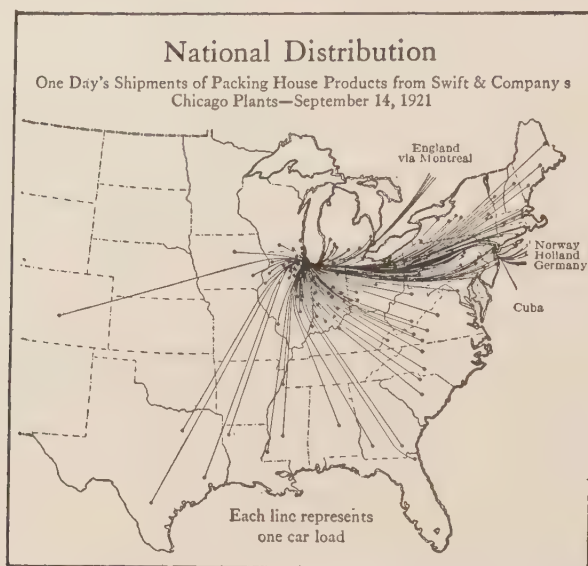


FIG. 238.—Density of traffic flow. (Courtesy of Commercial Research Department of Swift & Co.)

scale, such as thousands of persons per hour, which should be made large enough to show all important traffic, and yet it should be small enough so that there is room for the lines in the congested districts.

Geographical Distributions Where Something Other Than Area is a Factor.—When a distribution is to be shown where something other than area is a factor, such as number of automobiles per thousand of population, the density dot map cannot be used in the form just described. A method much in vogue at present is illustrated in Fig. 240 in which the states are drawn, not in proportion to their actual areas, but in proportion to their

LOS ANGELES
PASSENGER TRAFFIC FLOW
DURING AFTERNOON RUSH ON ALL LINES OF
LOS ANGELES RAILWAY AND ON CITY LINES
OF PACIFIC ELECTRIC RAILWAY

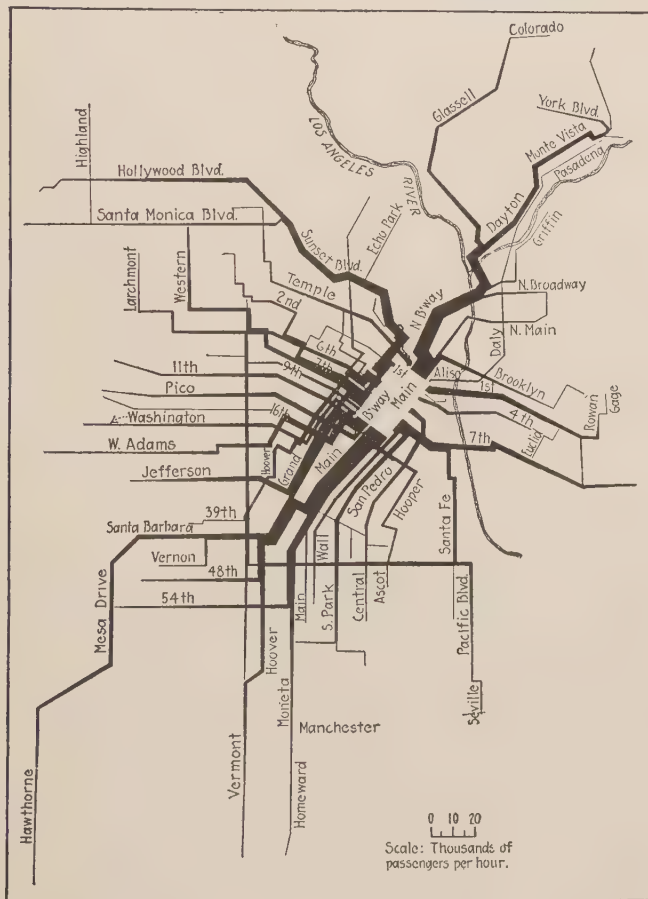


FIG. 239.—A traffic flow map. (Courtesy of Joe R. Ong.)

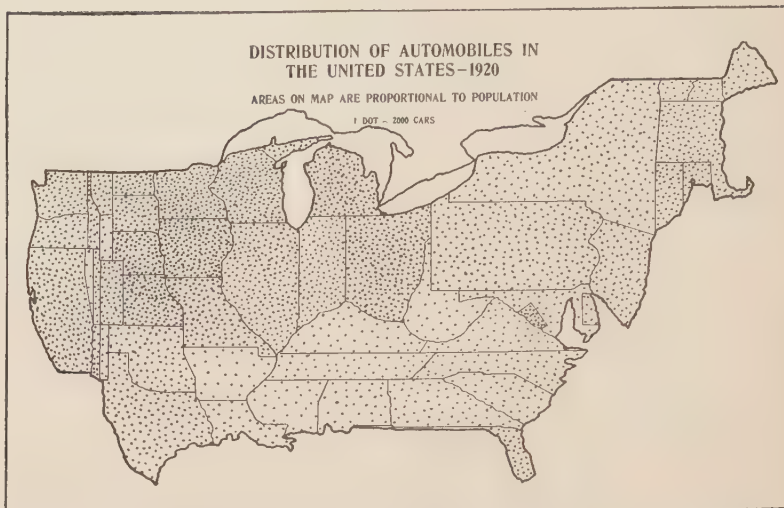


FIG. 240.—Showing the relation of population to automobiles by drawing the states in proportion to population. (Not recommended for general use.)

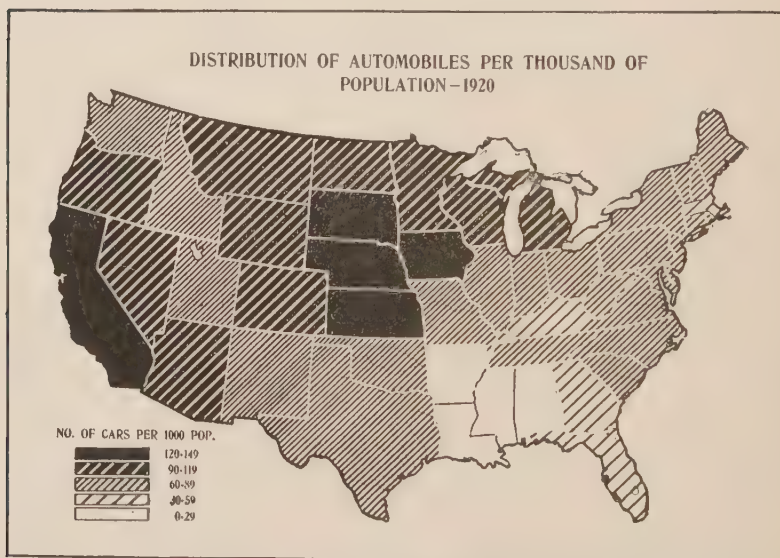


FIG. 241.—Another method of showing the relation of population to automobiles. Usually this method is to be preferred over the method shown in Fig. 240.

populations. Then the dots representing automobiles show their distribution in relation to population. This method of distorting the map, however, is not recommended for general use as it is quite likely to confuse the reader and leave an incorrect impression of sizes and distances. In fact, the relative positions of the states can be shown only approximately.

A better form of showing the data of Fig. 240 is the cross-hatched map illustrated by Fig. 241. This is not an absolute rule, however, since the two maps do not show quite the same thing. Figure 240 compares populations of states and shows the relative density of the automobile owning population but it does not show area, while Fig. 241 compares areas but does not show comparative populations. In accordance with the method previously explained in this chapter, it should be noted that the range of shading in Fig. 241 is graduated equally from white to black in five parts. The middle group is half black and half white. The second shading from the top is the negative of the second from the bottom. These shades vary by 25 per cent from one extreme to the other. That is, the bottom shade is 0 per cent black, the second is 25 per cent black, the third is 50, the fourth is 75, and the top is 100 per cent black.¹

Here, as in representing simple sizes, the cross-hatched map represents conditions as uniform within each district shaded and does not suggest gradual changes as well as the dot density map. In instances where abrupt changes actually occur, and where conditions within the areas are practically uniform, the cross-hatched map leaves correct impressions of the distributions. This, however, is not true of the actual distributions of the data of Fig. 241 in which changes in the distributions occur gradually and in this respect the distorted map of Fig. 240 has an advantage.

Use of Colors in Statistical Mapping.—The use of colors in statistical map work is limited. No scheme of representing different values by different colors can be used to give a visual impression that corresponds directly with the distribution of the data. No arrangement of red, orange, yellow, green, blue, and violet can give a clear impression of a graduated series of numbers. That is, for instance, green would not naturally impress a reader as representing a larger or smaller number than blue or red. Even if different intensities of the same color, graduated from

¹ Ideally, of course, 100 per cent black would represent the saturation point. In this map it is used merely to represent the highest class.

Comparisons of Geographical Distributions.—When it is desired to compare different geographical distributions for the same area, it is usually best to make different maps of the data as shown in Figs. 242 and 243. In such cases the maps should be drawn to the same scale and the same units should be plotted if possible. Because of the different units used in Fig. 242—cattle on one map and pounds on the other—the comparison is more difficult to comprehend than if the dot had been given the same value on both maps.

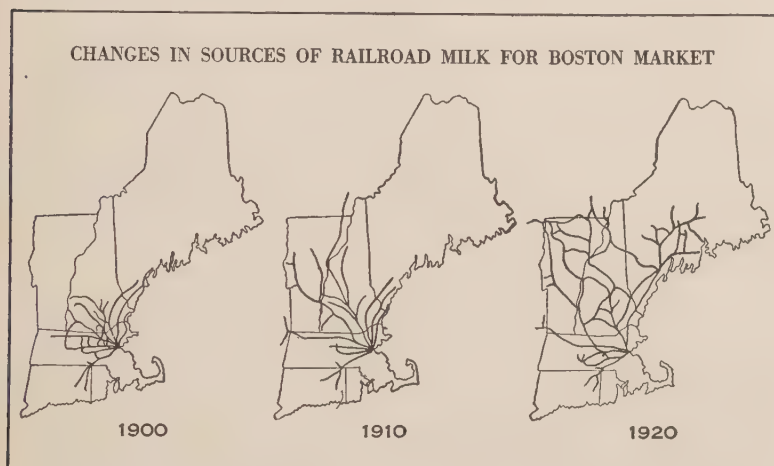


Fig. 243.—Comparing geographical distributions. (Courtesy of the Boston Chamber of Commerce.)

Figure 243 illustrates a good method of comparing the sources of Boston's milk by showing the actual distribution of the routes over which it is carried. In this chart no attempt is made to show the importance of the various sources.

Tables Should Accompany Maps.—As a guard against misinterpretation and hasty generalization, as well as an aid in understanding, maps should be accompanied by the data represented, either as separate tables or as figures recorded directly on the map. If the data are grouped in classes, only approximate frequencies can be shown, and if the groups are wide, the map may leave an impression different from one gained by reference to the actual data.

Since not all types of maps are equally well suited for all purposes, entirely different impressions may be given by different

methods of presentation if the concrete data are not given to serve as a check.

The Map-tack System.—There are many useful mechanical aids on the market for graphically showing geographical distributions. Chief among these are the mounted maps, tacks, pins, beads, cords, rings, tags, and flags, which are used together in various combinations in what is commonly known as the map-tack system.¹

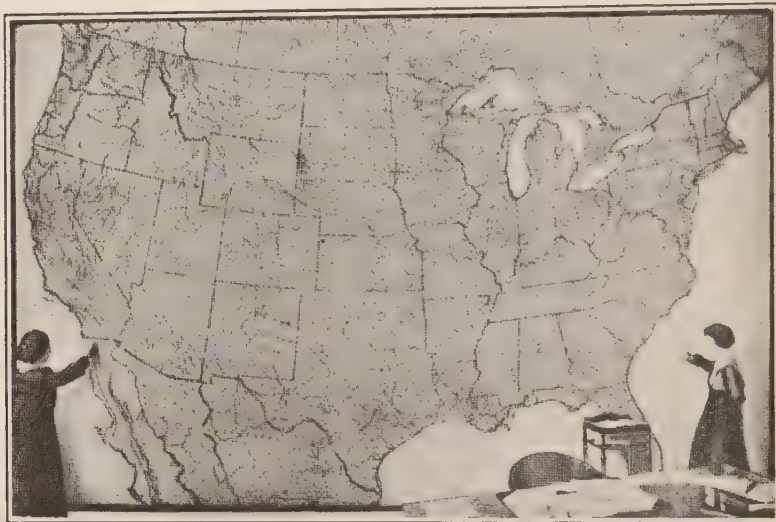


FIG. 244.—A large scale wall map. (*Courtesy of Rand McNally & Co.*)

In the routing of salesmen, in the designation of towns, branches, and agents, in the classification of purchases, and in analyzing competitors' strengths, the map-tack system is performing a very valuable service. This system of map making is very flexible. Tacks or other marking devices may be added, changed, or removed with but little inconvenience.

If tack maps are made for photographic reproduction, or for those to read who are not in close touch with the working of the particular map system used by the individual office, they should be made very simple, and according to the principles discussed previously in this chapter.

¹ Complete map-tack systems may be secured from Rand McNally, Chicago; National Map Co., Indianapolis; and Educational Exhibition Co., Providence. Inexpensive spherical headed pins may be obtained at 10-cent stores. If suitable colors cannot be obtained, the pin heads can be dipped in paints of the desired shades.

If, however, the map is made to be used by those who are in close touch with the individual system, it may be made more complicated, though care should be taken to keep the system simple enough to be understood easily by those who are to use it. In this type of map, disassociated facts may be successfully represented on the same map by different devices or different colors, because the readers are in close enough contact with the system to become familiar with their meanings quickly.

Tack maps vary in size from small hand maps to huge wall maps such as Fig. 244. These large maps sometimes are mounted in sections. It is important to have a map large enough, especially if there are congested districts to be studied. It is better to have a map too large than too small.

Map Mount.—Tack maps must be mounted upon a kind of board that will allow the pins to be pushed in to their heads without difficulty and yet hold each one securely. Unless the pin is

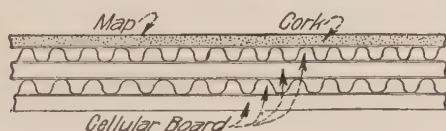


FIG. 245.—Cross-section of tack map mount. (Courtesy of the Educational Exhibition Co.)

pushed in to its head, it is easily knocked off the map, and as each pin is usually placed in position currently as the data are obtained, sometimes with no tabular record made, it cannot be replaced without a great deal of inconvenience.

Wood makes a satisfactory mounting for use with tacks if it is very soft, and made up, say, three-ply to prevent warping and cracking. A very cheap yet satisfactory mounting can be made with three or more layers of corrugated strawboard.¹ If the tacks are to be removed often, the strawboard should be faced with cloth or, better still, with cork. The structure of a form of ready-made mount which can be purchased is illustrated in Fig. 245. This mount consists of a cellular strawboard built up several

¹ These layers should be glued together with the ribs in the middle layer running crosswise to the others. The map should be mounted with paste. In order to equalize the shrinkage and tendency to warp, a piece of plain paper should be pasted on the back of the board at the same time the map is mounted. The finished map should be dried in a press or under weights. If the map is not printed in fast colors, it should be covered with a fixatif sprayed on with an atomizer before being wet with paste.

ply, the ribs in the alternate layers running crosswise to those in the other layers. On the surface of the strawboard is placed a thin layer of cork on which the map is directly mounted. While it is easy to push a pin into this kind of board, there is enough frictional resistance to hold it securely, and moreover the resilience of the cork closes the hole so that tacks may be pushed in many times at the same place without danger of working loose. If the pins are not to be removed, a single thickness of corrugated strawboard has been found to be quite satisfactory.

It is desirable to size and varnish the faces of tack maps as a protection against dampness and soiling.

Unless these maps are very small, they should be provided with some kind of a substantial frame. This is especially neces-



FIG. 246.—A map cabinet. (*Courtesy of Rand McNally & Co.*)

sary to prevent warping. Examples of the common forms of protection are the map cabinet (Fig. 246), the wall frame (Fig. 249), and the swinging display fixture (Fig. 253).

Map Marking Devices.—Of the devices used to indicate facts in the map-tack system, the most common are tacks, pins, beads, rings, flags, tags, stickers, and cords. Of these devices, the most important is the tack or pin.

Map tacks or pins are made with sharp needle points, and with glass, celluloid, and cloth-covered heads which are made in several hundred variations of sizes and colors. Cloth-covered tacks soil and fade more easily than do the glass and celluloid tacks. Figure 247 illustrates the use of spherical glass-headed tacks

and flat celluloid tacks for representing automobile sales data. Such a use of maps and tacks presents a picture of the distribution that is practically impossible from tables. The large celluloid tacks are surfaced so that numbers or other designations may be written upon them. Different distributions can be

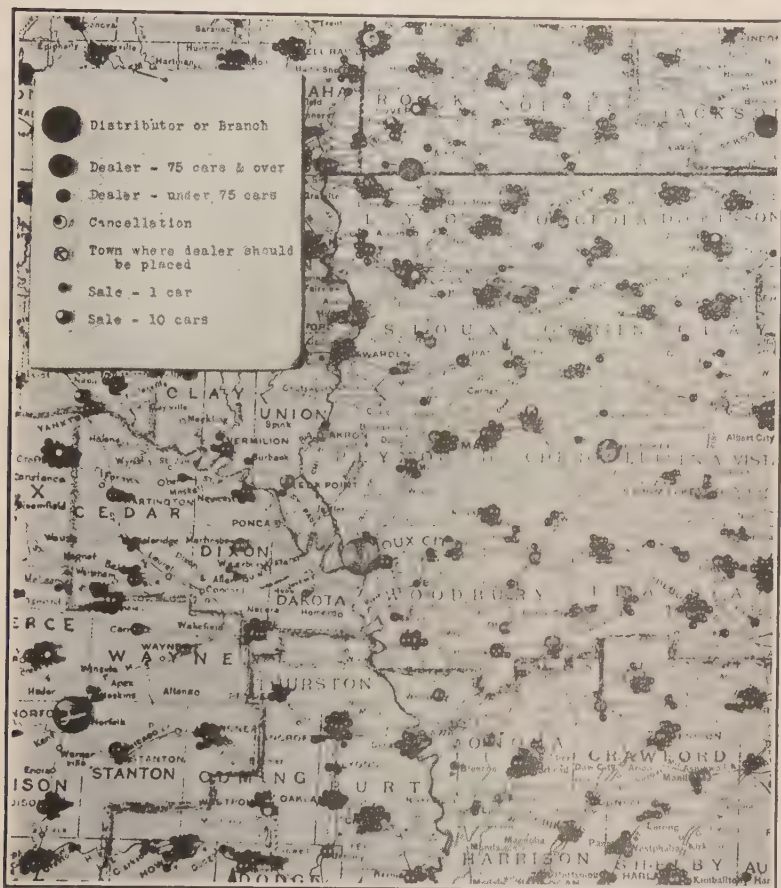


FIG. 247.—A tack map formerly used by the Willys-Overland Company. (Courtesy of Rand McNally & Co.)

presented clearly by using different-colored tacks, but different colors should not be used for presenting different degrees of value in the same distribution. For instance, one might show clearly the distribution of population with black tacks all of the same size, and the sale of automobiles by using white tacks all of the

same size, but from a graphic point of view it is not good practice to make a map allowing, say, one black dot to represent one automobile and one white dot ten automobiles. In this respect the map illustrated in Fig. 247 is faulty. The tack representing the sale of ten cars does not directly impress one as being ten times as large or important as the tack representing one sale. Looking over the area as a whole, it is difficult to pick out the areas of greatest density of sales because the visual impression left by observing, for instance, two of the large white-topped tacks is



FIG. 248.—Use of map cord. (Courtesy of Rand McNally & Co.)

weaker than that left by observing, say, nine of the smaller tacks. From a graphic point of view, the visual impression should correspond directly with the facts presented. Otherwise, very inaccurate impressions are likely to be gained. Thus, in Fig. 247 the map would be improved by using ten small pins instead of one of the larger white-topped pins. There would be no difficulty in making this change on the area shown in Fig. 247, but in areas of great density there would not be room for so many pins if plotted on the basis of one pin for each sale.

A method of showing more accurately distributions which vary from extreme sparseness to great density is the use of beads and

pins. In this method, the spheres are not spread over the area but are piled up perpendicularly from the map. Where only one dot appears, a map-tack is used. The larger values are represented with long pins strung with beads which are exactly like the heads of the tacks. If the long pins available for this purpose



FIG. 249.—International Harvester Company's tack map with card index system.
(Courtesy of Rand McNally & Co.)

are too short, steel wire can be used. When a large number of beads are used on a pin or wire, it is well to make every tenth bead another color so that it will be easy to read the number.

For supplementing map tacks and beads, other devices such as rings, tags, flags, and stickers are sometimes used to show addi-

tional information. These may bear notations or they may represent the information by their color or form.

When territories are to be marked off or routes are to be laid out, the lines can be made conveniently by using map cord as shown in Fig. 248. Any desired changes can be made very easily and without damage to the map simply by shifting the cord.

Washable maps can be secured which will allow marking and erasures. They may be marked with pencil, crayon, or pen, and erasures are made by washing as one would wash a black-board or slate.

Often it is desirable to have some means of tying the information of a map to the more complete and detailed information given in the various records of the firm. Figure 249 illustrates a method of using a card index system in connection with a pin map.

CHAPTER X

DUPLICATION, REPRODUCTION, AND DISPLAY

Since it is often necessary to have more than one copy of charts, it is important to give some consideration to methods of duplication and reproduction. The choice of the particular method to be used will depend largely upon the purpose of the chart. If, for instance, only a few copies are to be made for the use of a board of directors, the method may differ greatly from the one used to reach the public at large through a magazine.

Blueprinting.—The method most commonly used by engineers for duplicating drawings is that of blueprinting. A drawing intended for blueprinting is usually laid out on drawing paper, and then traced in ink upon tracing paper or tracing cloth which is translucent. If the drawing is not complicated it may be made directly upon the tracing cloth or paper.

Tracing cloth is usually preferred to tracing paper because it is more durable, more translucent, and more satisfactory to work upon. It is a fine cloth treated with a starch preparation. One side is glossy and the other is dull. Either may be used but most draftsmen prefer the dull side. To remove the traces of grease from the surface of the cloth, it should be rubbed with powdered chalk or pounce. This permits the ink to flow freely and evenly. To insure a good line, an opaque ink must be used. Colored inks often are not heavy enough to print well. To produce a light shading, thin colors or pencil tints are sometimes spread over the area instead of cross-hatching. This will print in very light blue. Care should be taken to protect tracing cloth from moisture as the starch preparation is highly soluble in water.

For an unimportant piece of work, charts are sometimes blueprinted from bond or ledger paper, but such prints do not always present a pleasing contrast and the watermarks show plainly. Any clear, white paper, however, may be made translucent enough to make a good print by oiling with "Nujol" or by applying "transparentizing solutions" which are handled by most dealers in drawing supplies.

If typewritten material is to be blueprinted, a good ribbon should be used and a good carbon paper should be placed in the machine to face the back of the tracing paper. With the ribbon ink on one side of the paper and the carbon ink on the other a fairly opaque letter is made.

Blueprinting, after the original is made, is a simple process. A piece of sensitized blueprint paper, with the tracing in front of it, is exposed to the sun or electric light for a few seconds or minutes depending upon the rapidity of the paper. A chemical action takes place in the exposed parts which renders the coating insoluble. This turns blue when the paper is washed in a water bath, and the soluble parts, which were protected from light, wash out leaving white lines. In an overexposed print, the blue color may be intensified and the white lines cleared by dipping in a bath containing potassium bichromate or sodium bichromate (1 or 2 ounces of crystals per gallon of water) and again washing. Blueprint paper is usually secured ready sensitized. It may be bought in various widths, weights, and speeds of printing. When fresh, it is of a greenish-yellow color. However, it deteriorates rapidly, especially upon exposure to air, light, or moisture, and cannot be used at all after a comparatively short time.

The ordinary blueprint reverses the black and white of the original. That is, the black lines on the tracing are white on the blueprint. This is the cheapest and most common form of print, though there are several blue-line and black-line papers on the market by which positive copies can be made directly from the tracing. In another method of duplicating positive prints, a negative is first made on Van Dyke paper.¹ This is a thin paper which turns dark brown on exposing and fixing. Then from these negatives blue-line prints with white backgrounds can be made from blueprint paper, and brown-line prints can be made from a special paper prepared for the purpose.

The equipment used in blueprinting varies all the way from almost nothing to the machine illustrated in Fig. 250. In this machine the tracings and paper are fed through rolls and the printing, washing, and drying are done in one operation. Electric lights are used when the operator wishes to be independent of the sunlight. Frames mounted on carriers so that they may be run out of a window are commonly used when the exposure is made to the sun. Good prints may be made from small tracings

¹ This paper is given different names by different companies.

simply by holding them to the sunlight against a window pane. By keeping a small quantity of blueprint paper on hand, it often happens that copies of tables, graphs, or printed matter may be made in this simple way much more quickly and cheaply than they can be copied by hand or on a typewriter.

If an exact duplicate is wanted of a tracing, there are firms that do this kind of work by a gelatine process in which a special

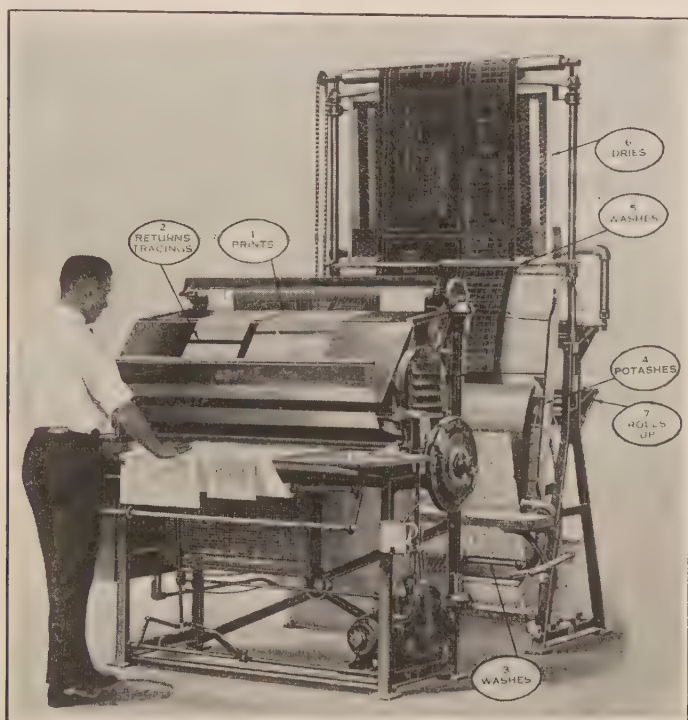


FIG. 250.—Electric blueprinting machine with washing and drying equipment.
(Courtesy of the C. F. Pease Co.)

matrix print is made in a blueprint outfit and transferred to a gelatine-surfaced table. The impression is then inked and prints pulled from it which can be used for making blue prints equal in every respect to those made from original tracings.¹

Photostat negatives oiled with "Nujol" or any purified paraffine oil, and well blotted, make good negatives for blueprinting.

¹ Two companies which duplicate tracings are the Lithoprint Co., 41-43 Warren St., New York City and the Rexim Co., Inc., 724-26-28 Cherry St., Philadelphia.

Unless one regularly has a large amount of printing, the work is usually sent out to commercial blueprinting houses. This is cheaper than maintaining equipment unless the amount of work is large.

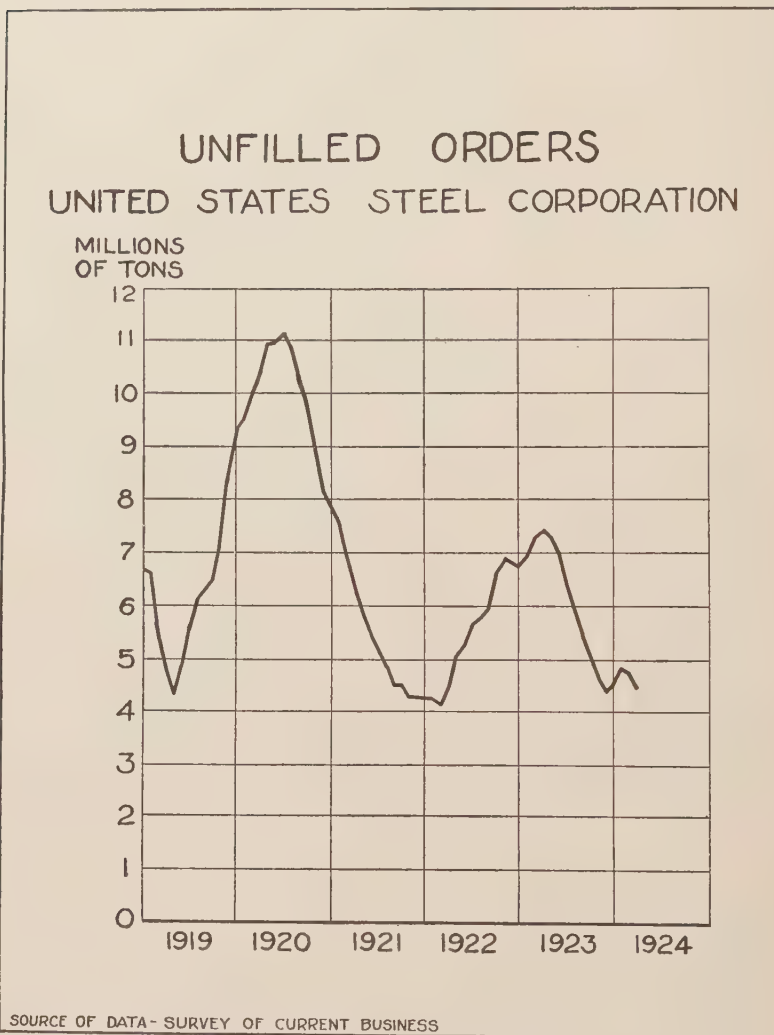


FIG. 251A.—Drawing—full size.

Drawing for Photographic Reproduction.—Much reproduction work is done by methods which include some use of photographic

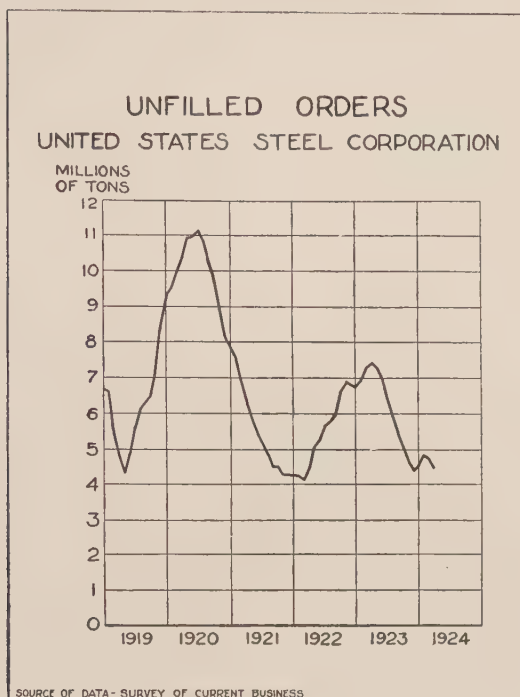


FIG. 251B.—One-third reduction of Fig. 251A.

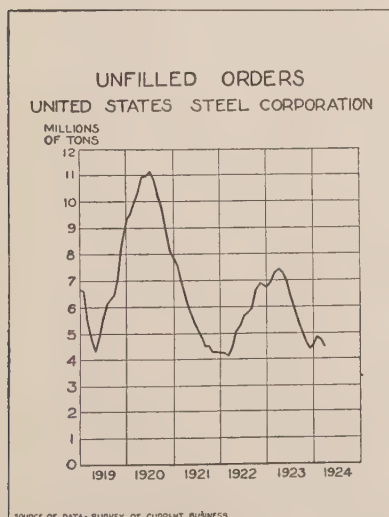


FIG. 251C.—One-half reduction of Fig. 251A.

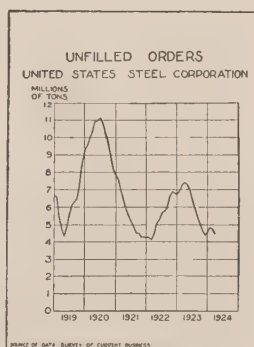


FIG. 251D.—Two-thirds reduction of Fig. 251A.

processes. This kind of work includes that of the photostat and the many kinds of plates or "cuts" for printing the illustrations in books, magazines, newspapers, circulars, and other publications.

Drawings for photographic reproduction should be made on smooth white paper or tracing cloth and in black ink for the best results. Green, red, and yellow inks will photograph (as black) but pure blue will not show at all in ordinary light. Usually a chart that is to be reproduced by a photographic process is made somewhat larger than the reproduction. The reduction tends to eliminate minor irregularities, and gives a much more finished appearance than is possible when charts are reproduced in the

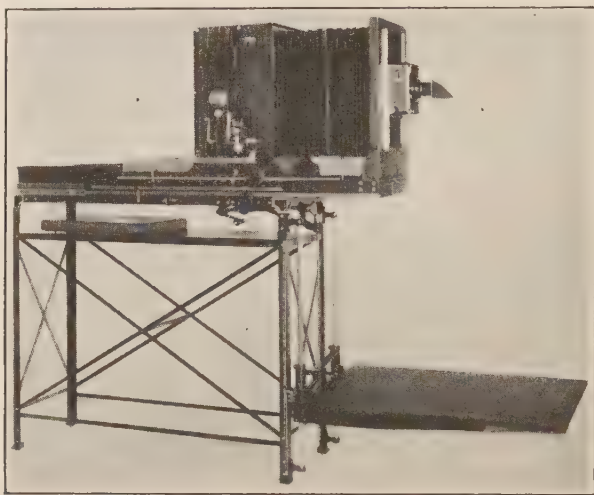


FIG. 252.—Number 1 photostat. (*Courtesy of the Photostat Corporation.*)

exact size of the original. Figure 251 shows how a full-sized reproduction looks when it is reduced one-third, one-half, and two-thirds linear. Drawings are sometimes made many times the size of the reproduction, but as a rule originals made two or three times (linear) the size of the reproductions will give the best results.

When drawing for reproduction, particular attention should be given to the widths of lines. A two-thirds reduction (linear) of course reduces the width of the line to one-third of its original width. A very helpful aid in judging the appearance of the reduction is the reducing glass, which is a concave lens mounted like a reading glass.

On drawings for photographic reproduction, irregularities and errors may be corrected by painting out with Chinese white (water color). Lettering may be printed with type on strips of paper which may be pasted in their proper position on the drawing. If the edges are carefully painted with Chinese white and care is used in photostating, the edges of the pieces pasted on will not show. Or, if a cut is made, any such lines will be tooled out by the engraver when the cut is finished.

The Photostat.—There are several direct photographic methods for reproducing drawings but the most commonly used is the photostat (Fig. 252). In this process the sensitized paper is exposed to the chart through certain lenses and mirrors so that by a direct exposure a positive print of white lines on a black background can be secured. This method is especially useful for copying charts or printed matter from reports, books, or magazines. When only a few copies of a chart are to be made, this method is quite economical. Firms which have a large amount of copying to do usually have their own outfits, while others find it best to hire it done on the outside by commercial photostating firms.

Cuts or Plates for the Press.¹—When charts are reproduced by the printing press, plates or “cuts” are made of the originals by various photo-mechanical processes.

Drawings, such as charts, which are black-and-white copy with no gray tones, are commonly reproduced by cuts known as *line plates* or *line drawings*. These cuts are usually made by the process known as zinc etching, in which the drawing is photographed and a positive is printed on a sensitized zinc plate which is etched (corroded) with acid, leaving the protected parts or lines in relief. This is the cheapest and most generally used method of reproducing charts for the press. Care should be taken to see that the drawings are well made since all imperfections are, of course, reproduced.

A method of making cuts, which does not require a carefully drawn original, is that of wax engraving. In this process the engraver does the work of the draftsman and the lettering is put in with type. Thus, a very good cut can be made from a rough pen or pencil sketch. The chief disadvantage in using wax cuts at the present time is the high cost of engraving them.

¹ For a good discussion of this subject see SMITH, HERBERT W., “Picture Plates for the Press.” *University of Missouri Bull.*, Vol. 22, No. 28.

Another kind of cut, occasionally used in connection with graphic presentation, is made by the half-tone process, in which a negative is made through a screen which breaks up the shades into series of dots of different sizes. The eye blends the black tone of the dots with the white background of the paper and gives an impression of a gray tone. For different kinds of paper, screens of different fineness are used, varying from sixty lines to the inch for rotary press newspaper work to one hundred and fifty lines to the inch for the finest enamel papers. The most satisfactory half-tone illustrations are made from photographs or wash drawings. Photographs on glossy paper produce the best results.

Ben Day Shading.—The high cost of color printing prohibits its use for most purposes. When areas must be contrasted, usually some form of black-and-white shading or cross-hatching is used. This may be drawn by hand or it may be produced mechanically by the Ben Day process. The Ben Day shade is printed mechanically either on the plate by the engraver, or on the original drawing from an inked gelatine surface and rubbed on with a stylus. Most good engravers can do Ben Day work. The particular shades to be used are selected by the maker of the chart from a catalogue and designated by numbers on the drawing. An example of a chart on which Ben Day work is used is shown in Fig. 162 on page 125.

Mimeograph and Hectograph.—The methods of the mimeograph and hectograph are quite generally understood but may be mentioned here in connection with reproducing charts.

The mimeograph is probably the most widely known of the stencil duplicators. In this process a stencil is made through which a special ink passes to the paper. The original drawing is usually made in India ink on paper and then traced on the stencil over a specially prepared table with a ground glass top under which an electric light is mounted. Once the stencil is made, any ordinary demand for duplicates can be met easily and quickly. The stencil can be removed from the machine and used, when it is wanted, several times before it is destroyed.

In the hectograph method, the chart is drawn on paper with special hectograph ink. This is transferred to a gelatine pad simply by moistening the surface of the pad and placing the drawing face down upon it. Then papers placed in contact with the pad receive duplicates of the original. Colored hectograph

drawing inks are available. This method is simple and inexpensive and is very satisfactory when only about fifty copies are wanted. The original, however, can be used only once if the pad is charged to make many copies. Probably the most widely used machine of this type is the "Ditto."¹

In using either the mimeograph or the hectograph, typewritten material may be duplicated on the same sheet with the charts. Other similar methods may be suggested by referring to the advertisements in business magazines.

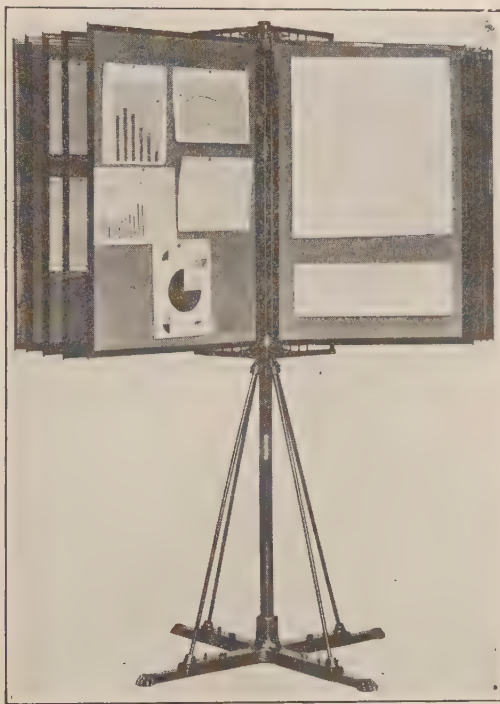


FIG. 253.—Fixture with swinging leaves for displaying charts. (Courtesy of *Multiplex Display Fixture Co.*)

Lantern Display.—It is often desirable to present a chart or table to groups in such a manner that various features may be pointed out directly and discussed. For this purpose, wall charts serve very well but they are expensive to make. Lantern slides are much cheaper and are very satisfactory for showing charts

¹ Made by Ditto, Incorporated, Chicago.

to a large number of persons. If the group is small, however, the card records themselves may be presented by means of the reflectoscope. This method has a distinct advantage over slides for small groups as it is much less expensive and may be kept up to date much more easily. If records are kept on 4- by 6-inch or 4- by 12-inch cards, the very latest information or data on file may be reflected on the screen for use in meetings such as those of department heads or boards of directors.

Display Fixtures.—If charts are to be of the greatest service, it is important to have them readily available. Current charts should not be hidden away in the files but should be where they may be referred to instantly and with very little effort. One of the best methods of handling such charts is to mount them on display fixtures such as the one illustrated in Fig. 253. There are many makes and types of these fixtures and many suggestions for display will be found by consulting the catalogues of the various fixture companies who advertise in business magazines.

INDEX

A

- Abscissas, 86
- Accuracy in charting, 37
 - in using drawing instruments, 5
- Aggregates, comparison of, 43
 - areas, 44
 - bars, 44
 - circles, 44
 - component parts, 73
 - geographical distributions of, 189
 - geometric forms, simple, 43
 - made up of components, compared, 75
 - bar chart, 76
 - size, 42
 - volumes, 44
- Analysis, charts in, 3
- Appeal to mind through eye, by charts, 1
- Area, aggregates, compared by, 44
 - basis for pictorial comparison of size, 45
 - between curves shaded to show component parts, 123
 - circular, to show sizes, 183, 184, 188
 - compared, 44, 181, 182
 - geographical, and distances compared, 181
 - of circles proportional to numbers, 44
 - representing sizes for, 186
 - shaded, designating, 121, 122
- Arithlog paper, 132
- Arithmetic and logarithmic scales, 131, 180
 - curves compared, 139
- Arithmetic division of given space, 10
 - progression, 132

- Arithmetic scale (see *Scale arithmetic*).
 - time curve, 87-89
 - vertical scale, 86
- Axes, coordinate, 86

B

- Bar charts, complicated, 70
 - component, 68-71, 77
 - pictorial, 73, 74
 - vertical with scale, 68, 69
 - designation of bars on, 56, 57
 - frequency, 161
 - horizontal, 49, 50, 84
 - for comparing groups, 62
 - for showing time series, 84
 - in comparing percentage distribution of component parts of aggregates, 76
 - mechanical, 64
 - scales, 52
 - designating, 54
 - used correctly, 52
 - simple, 49, 50
 - tabulation of data on, 56, 57
 - time series, 84, 85
 - vertical, 50
 - used to show time series, 84
- Bars (see also *Bar charts*).
 - aggregates compared by, 44
 - choice between, and curves, 102, 120
 - compared with areas and volumes, 44
 - component, connected by lines, 121
 - errors in using, 77, 78
 - with no intervening spaces, 122
 - data blocked in, bad practice, 57, 58
 - geographical distribution of sizes shown by, 190

- Bars, horizontal, comparing groups
 with, 62
 lengthening, 12
 sizes at geographic points shown
 by, 182
 used on logarithmic scale, 142, 143
 vertical, comparing groups with,
 61
 Base line of logarithmic scale, 136
 Beginners, common faults in ruling
 lines, 16
 Belts, on a component time chart,
 123
 Ben Day shading, 220
 Blueprinting machine, 215
 process, 213-215
 Border line of chart, 41
 Bow instruments, 14, 15
 dividers, 14
 pen, 14
 pencil, 14
 Braddock lettering triangle, 24
 Bristol board, 6
 Brushes, water-color, 18
- C
- Cabinet, map, 208
 Capitals, 26, 28
 inclined, 27, 28
 vertical, single-stroke, 26
 Caption of scale, 54, 87
 Card-index system for maps, 211, 212
 Cards placed end to end to form a
 continuous chart, 37
 placed one above another for
 comparing seasonal varia-
 tions, 37
 Cement, rubber, 19
 Change, rate of (see *Rate of change*).
 Changes, size, curves of, 86
 of single variable, 81
 within the whole, shown by simple
 method, 124
 Charting, accuracy in, 37
 forms, 35
 paper, 6
 Charts, accuracy, 37
 an aid to memory, 2
- Charts, appeal to mind through
 eye, 1
 appearance, 41
 bar (see *Bar charts*).
 border lines, 41
 break at bottom, 89
 circle, 65
 color on, 39
 component, common errors in
 making, 78
 time, designation of belts, 123
 continuous, 34
 data tabulated on, 101
 development of, 2
 displaying from fixture with
 swinging leaves, 221, 222
 enlarged, 34, 35
 graphic, in business, 1-4
 in analysis, 3
 in business, 3
 logarithmic (see *Logarithmic
 charts*).
 margin, 41
 pictorial (see *Pictorial charts*).
 pie (see *Pie chart*).
 planning, principles of, 33-41
 printed forms, 35
 proportions, 34
 purpose, 33
 ratio, 132
 semilogarithmic, 132
 showing component parts of com-
 ponent parts, 74, 75
 size, 34
 standardization, 36, 37, 111
 statistical, technique of drawing,
 5-32
 tabulation of data on, 101
 time (see *Time charts*).
 saved by, 3
 titles, 38
 varieties, 45, 49, 63, 65, 77
 visual appeal, 1, 2
 wall, 34, 35
 mechanical, 63, 64
 Checking lists, 78, 79, 128, 129
 Chinese white, 31
 Circle charts, 65
 maps, 183-185

- Circles, comparison of, 44
 to show distances from a central point, 185
 with areas proportional to numbers, 44
 with diameters proportional to numbers, 44
- Class intervals, cumulative frequency curves with unequal, 169
 designation of, 158
 in frequency curve, 156, 158, 169
 limits, designation of, 159, 160
- Coefficient of correlation, 174
- Colors, in statistical mapping, 203
 on charts, 39
 water, 18
- Column diagram, relation between broken curve and, 163
- Combination "less than" and "more than" cumulative frequency curve, 167, 168
- Compasses, 12
- Component bars (see *Bars, component*).
 chart (see *Charts, component*).
 curves, to show percentage distribution of several components, 126, 127
 parts, aggregates, geographical distribution of, 189
 areas between curves shaded to show, 123
 of aggregates compared, 65, 73, 75-77
 of component parts, 75
 of time variable, 120
 pictorial forms for showing, 72-74
 time chart, belts on, 123
- Compressed letters, 24
- Continuous chart, 34
 series in frequency graph, 159
- Coordinate axes, 86
 lines (see *Lines coordinate*).
 paper, 35, 36
- Coordinates, 86
 polar, 99, 100
 rectangular, 99
- Copying devices, 20, 21
- Cord, map, 210
- Correlation, 172-180
 coefficient of, 174
 curves, compared, 177-179
 logarithmic, 180
 primary, 172, 173
 secondary, 173, 174
- Crayons, 18
- Cross-hatched maps, 203
- Cross-hatching, optical illusions in, 20
- Curve pen, 15
- Curves, areas between shaded, to show component parts, 123
 arithmetic and logarithmic compared, 139
 bell-shaped, 164
 broken, relation between column diagram and, 163
 brought together by percentage relatives, 149
 choice between bars and, 102, 120
 component, to show percentage distribution of several components, 126, 127
 confusion of, 109, 110
 correlation, compared, 177-179
 logarithmic, 180
 designating, 108, 109
 differentiating, 107, 108
 fitting to scattered points, 177
 French, 9
 frequency (see *Frequency curves*).
 instruments, 8, 9
 irregular, 8
 logarithmic (see *Logarithmic curves*).
 number of, 108-111
 of percentage relatives, 90
 of size changes, 86
 parallel, 141
 popular adaptation of, in magazine advertising, 40
 position of, 176, 177
 shifted to aid comparison, 146
 smoothed and broken, 95-98, 161-166, 176
 when showing continuous series, 175

- Curves, step, 98, 99
 time, arithmetic, 86-120
 weight of, 93
 Cut-out letters, 31
 Cuts, for press, 219, 220
- D
- Data, cumulative, 97
 non-cumulative, 96
 tabulation on chart, 101
 bar charts, 56, 57
 Decreases, shown with increases, 59, 60
 Density, indication by dots, 195, 196
 maps (see *Maps, density*).
 of linear magnitudes, 199
 of traffic flow, 200, 201
 shown by shading, 198
 Deviations, standard, 152, 153
 Diameters, of circles proportional to numbers, 44
 Discrete series, 159
 Display, 213-222
 fixtures, 221, 222
 Distances, and geographical areas compared, 181, 182
 from a central point shown by circles, 185
 radial, 185
 vertical, only to be compared, 140
 Distortion caused by different scales, 112
 Distribution, frequency, 155-171
 compared, 169, 170
 geographical, 181-212
 of aggregates showing their component parts, 189
 of sizes shown by bars, 182, 190
 where something other than area is a factor, 200
 percentage, comparison of, 77
 size, 181
 unlike time, compared, 144, 145
 Dividers, 13, 14
 proportional, 14
 Division of a given line, 11
 of a given space, 10, 11
 arithmetic, 10
 irregular, 11
 logarithmic, 10
- Dot maps, 183, 184, 188-190, 195-197, 202, 204, 209
 Dots, grouped to show sizes, 190
 indication of density by, 195, 196
 Dotting pen, 15
 Drawing board, 7
 equipment, 5, 6
 paper, 6
 reduction of, 216-219
 Drop pen, 15
 Duplication, 213-222
- E
- Equipment, selection of, 5
 Erasers, 17
 Erasing shields, 17
 Errors in comparing size, 59
 in component bars, 77, 78
 charts, 78
 in lettering, 29
 in shading, 127
 in using common scales, 106
- F
- Fluctuations, relative, compared, 141
 Forecasts, indication of, 119
 Frequency bar charts (see *Bar charts*).
 curve, 155
 bell-shaped, 164
 broken, 160-165
 compared on percentage basis, 169-171
 cumulative, 166-169
 equal class intervals in, 158
 "less than" cumulative, 167
 "more than" cumulative, 168
 normal, 164
 simple, 165
 smoothed, 160-165
 unequal class intervals in, 157, 169
 distributions, 155-171
 compared, 169-171
 graphs, class intervals, 156
 continuous series in, 159
 scales, 156
 series, and historical, 174
 distinguished from other series, 174

G

- Gains, shown with losses, 59, 60
- Geographic distances compared, 182
 - distributions, of time variables, 184
 - compared, 204, 205
- Geometric forms, for simple comparison of aggregates, 43
 - used in size comparisons, 44
- Geométic progression, 132
 - scale, 132
- Glass boards, 20
 - reducing, 218
- Glue, 19
- Gothic lettering, 22, 29
- Graphic charts in business, 1-4
- Graphs, frequency (see *Frequency graphs*).
- Groups, compared with vertical bars, 61
- Growth, rate of, 131
- Guide, lettering, Wrico, 24
 - lines, 55
 - for lettering, 26
 - scale, 53

H

- Half-tone process, 220
- Hectograph, 220, 221

I

- Illusions, optical (see *Optical illusions*).
- Increases shown with decreases, 59, 60
- Ink bottle holder and pen filler, 17
 - colored, 18
- India, 17
 - pen blade corroded by, 16
 - too much, in lettering, 23
- Instruments and materials, 6
 - compasses, 12
 - copying device, 20, 21
 - curves, 8, 9
 - dividers, 13
 - drawing board, 7
 - paper, 6

- Instruments and materials, erasers, 17

- erasing shield, 17
- ink, 17
- pencils, 7
- pens, 15
- protractor, 12
- scale (statistician's), 9
- section liner, 20
- triangles, 8
- T-square, 7
- water-color brushes, 18
- water colors, 18
- Intervals, class (see *Class intervals*).
- Items, order of, 61

L

- Lantern display, 221, 222
- Lettering, 21-32
 - Braddock, triangle, 24
 - errors in, 22, 29
 - free-hand, 25
 - Gothic, 22, 29
 - guide lines, 26
 - Wrico, 24
 - hand, 21, 22, 25-28
 - instruments, 22-24
 - legibility in, 21
 - mechanical, 21, 31
 - modern roman, 22, 30
 - optical illusions in, 25
 - pens, 22, 23, 24
 - requirements, 21
 - single-stroke, 25-28
 - capitals, 26, 28
 - lower-case, 27, 28
 - slanting, 27
 - speed in, 21
 - styles, 22
 - triangle, 24
 - vertical and horizontal, 51
- Letters, bold-face, 25
 - capital, inclined, 27, 28
 - compressed, 24
 - cut-out, 31
 - extended, 24, 25,
 - gummed, 31
 - light-face, 25
 - lower-case, 27, 28

- Letters, lower-case, inclined, 28
 vertical, 27
 outline, 29
 proportions of, 27
 spacing, 30
- Line, basis for pictorial comparison
 of size, 45
 division of a given (see *Division*).
 drawings, 219
 100 per cent, 93
 plates, 219
 zero, logarithmic chart, 136
- Lines, beginners' faults in ruling, 16
 coordinate in background of chart, 94
 number of, 94
 emphasized, 93
 faults in ruling, 16
 guide, 55
 for lettering, 26
 scale, 53
 weight of, 93
- Logarithmic charts, 132
 compared, 145
 designations on, 143
 of five decks, 137
 of two decks, 136
 scale units, 143
 slope of curves, 132, 136
 standardization, 146
 unlike time distribution compared on, 144, 145
 zero line, 136
- curves, compared with arithmetic, 139
 rate of change indicated by
 slope of, 138
 scales, 131
 shifted for ease of comparison, 146
 simplicity of, 131
 smoothed, 138
- paper, letter-size, 135
- scales and arithmetic, 131
 bars used on, 143
 no base or zero line on, 136, 137
 standardization of, 146
 use on correlation charts, 178
- Losses shown with gains, 59, 60

M

- Magazine advertising, popular adaptations of curves in, 40
- Map cabinet, 208
 cord, 210
 mount, 207
 tack, mount, cross-section, 207
- Maps, card index system for, 211, 212
 circle, 183-185
 cross-hatched, 202, 203
 density, 195-204
 of United States, on county basis, 196
 distorted to show size, 191, 202
 dot, 183, 184, 188-190, 195-197, 202, 204, 209
 marking devices, 208
 outline, 181
 shaded, 192
 statistical, but not graphic, 187
 tables should accompany, 205
 wall, large size, 206
 washable, 212
- Marking pens, automatic, 24
- Mimeograph machine, 220
- Minus and plus quantities, 95
- Months, designating by quarters, 90, 91
- Mount for map, 207
- Mounting, 19, 20
 cold, 20
 hot, 19

N

- Needle point, adjustment of, 13
- Numbers, circles with areas and diameters proportional to, 44
 method of representing, 183
- Nujol, 213

O

- "Offset" printing process, 36
- One hundred per cent line, 93
- Optical illusions, 20, 25
 in cross-hatching, 20
 in lettering, 25
- Order of items, 61

Ordinates, 86
Origin, point of, 86

P

Pantograph, 21
Paper, arithlog, 132
 coordinate, 35, 36
 drawing, 6
 logarithmic, 135
 semilogarithmic, 132, 133
Paste, 20
Payzant lettering pens, 23
Pencils, 7, 14
Pens, 14, 15, 22-24
 automatic marking, 24
 blade, corroded by ink, 16
 bow, 14
 curve, 15
 dotting, 15
 drop, 15
 filler and ink bottle holder, 17
 lettering, 22
 payzant, 23
 railroad, 15
 ruling, 15
 writing, 22
Percentage basis, for comparing frequency curves, 169-171
 distributions, comparison, 76, 77
 of components, shown by component curves, 126, 127
 relations, of two components, 125
 relatives, 88, 147
 curve of, 90
 scale, 89
 to bring curves together, 149
 simple method, 150, 152
Percentages, and actual sizes, 122
 shading to show, 193
Photostat, 218, 219
Pictorial charts, 40, 45
 for popular presentation, 39, 40
 of time series, 81-83
 pie, 67, 68
Pie chart, 65
 correctly and incorrectly made, 65, 66
 percentage distribution of component parts shown by, 76
 pictorial, 67, 68

Plates for press, 219, 220
Plus and minus quantities, 95
Points in broken series connected by
 straight line, 176
 plotted, position, 92
 showing when curve is freely smoothed, 165
Population comparisons, 46
Presentation, necessity for clear, 1
 popular, 47
Printing press, drafting room, 31
 hand, 31
Progression, 132
 arithmetic, 132
 geometric, 132
Protractor, 11, 12

Q

Quantities compared, 140
 plus or minus, 95
Quarters, months designated by, 90, 91

R

Radial distances, 185
Railroad pen, 15
Rate of change, compared, 141
 common units unnecessary in comparing, 144
 constant, represented by straight line, 138
 in more than one variable, 138
 in time variable, 130
 indicated by slope of logarithmic curve, 138
 of growth, 131
 percentage method of showing, 150
Ratio chart, 132
Reducing glass, 218
Reductions, drawing with, 216-219
 photographic, 216-219
Reproduction, 213-222
 drawing for, 216
Roman lettering, 22, 30
Rubber cement, 19
 stamps, 31
Rulers, flexible, 9

Ruling pens, 15, 16
 blades, 17
 Ruling, semilogarithmic, 133

S

Sales, branch and main store compared, 139
 Scales, 9, 52, 54
 arithmetic, and logarithmic, 131, 180
 differences in sizes shown on, 131
 vertical, 86
 bar chart (see *Bar charts*).
 caption, 54, 87
 distortion caused by different, 112
 errors in using common, 106, 107
 guide lines, 53
 horizontal, 89
 omission is not represented graphically, 93
 in arithmetic and geometric progression, 132
 juggling of, 112
 logarithmic (see *Logarithmic scale*).
 method of including when no background of guide lines used, 55
 of frequency graphs, 156
 percentage relative, 89
 position for, 54
 size comparisons, 104
 statistician's, 9
 time, 116
 double, 113
 unit of, 52, 94
 bar charts, 54
 logarithmic charts, 143
 vertical, arithmetic, 86
 Section liner, 20
 Semilogarithmic chart, 132
 paper, 133
 ruling, 133
 Series, broken, 159
 points in connected by straight lines, 95, 96, 176
 continuous, curve smoothed when showing, 97, 98, 175

Series, discrete, 159
 frequency (see *Frequency series*).
 time (see *Time series*).
 Shading, areas between curves to show component parts, 123
 Ben Day, 220
 components, 128
 density shown by, 198
 differences shown by, 96, 118, 119
 errors in, 127
 method graduated from 0 to 100, 193
 percentages shown by, 193
 to show distances from a central point, 186
 sizes represented by, 192, 193, 198, 202
 variations emphasized by, 96
 Sizes, actual, and percentages, 122
 changes in, curves of, 86
 of single variable, 81
 comparison of, 42-79, 140
 difficult, 46
 errors in, 59
 geometric forms in, 44
 poor presentation, 49
 scales of, 104
 unit necessary in, 63, 64
 differences between time variables compared, 102
 shown on arithmetic scale, 131
 distribution of, 181
 dots grouped to show, 190
 for areas, representing, 186
 for geographic points and areas shown by circular areas, 183, 184, 188
 by bars, 182, 190
 of aggregates, compared, 42
 of chart, 34
 pictorial comparison, 42, 43
 on area basis, 45
 on line basis, 45
 represented by shading, 192
 shown by distortion of map, 191, 202
 Slope of curves, in logarithmic chart, 136
 Space, division of a given (see *Division*).

Spacing, 30
 unequal, for years, 92
Speed, in lettering, 21
Stamp-o-graph process, 31
Standard deviations, 152, 153
Standardization of charts, 36, 37, 111
 of logarithmic scales, 146
States, drawn in proportion to figures represented, 191, 202
Statistician's scale, 9
Step curve, 98, 99

T

Tables should accompany maps, 205
Tack map, 209
 with card index system, 211
Time chart, component, belts on, 123
 pictorial, 82
 pictorial component, 124
Time curve, arithmetic, 87-89
 distributions, unlike, compared, 144, 145
 periods, 114
 comparison of, 82, 114
 saved by use of charts, 3
 scales, 116
 double, 113
 series, 80-154, 174
 bar charts, 84-86
 but not historical, 175
 compared, 148
 continuous, 97
 historical, 80-154, 174
 methods of comparing, 148
 pictorial chart of, 81-83
 secondary correlation between, 173
 vertical bar chart for showing, 84
 variables, component parts, 120
 differences in size compared, 102
 geographic distribution of, 184
 rates of change in, 130
Tinting, 18

Titles, necessity for clear, 39
 pasting on printed, 31
 position of, 38
 size of, 38
Traffic flow, density of, 200, 201
Transparentizing solutions, 213
Triangles, 8
 Braddock lettering, 24
T-square, 7
Typewriter, 32

U

Unit, common, unnecessary in comparing rates of change, 144
 necessary in size comparison, 63, 64
 of scale, 52, 94
 logarithmic charts, 143

V

Values, positive and negative compared, 106
Variables, 173
 dependent, 173
 independent, 173
 time (see *Time variables*).
Variations, negative and positive, 95
 range of, bars, 114
 shading between curves, 113
 shading to show, 96, 118, 119
Volumes compared, 44

W

Wall charts (see *Charts*).
Water colors, 18
 brushes, 18
Wax engraving, 219

Z

Zero lines, 52, 53, 87, 93, 105
 on arithmetic curve, 105
 on horizontal scale, 156
 on logarithmic chart, 136

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